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ROYAL ASTRONOMICAL SOCIETY.

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SIR DAVID GILL, K.C.B., F.R.S., PRESIDENT, in the Chair.

Frederick William Bateman, Westergate, 20 Corfton Road, Ealing, W.;
John Brittain, Lecturer on Astronomy, Local School of Mines, Ballarat, Victoria, Australia;
R. Burdon-Martin, Mathematical Master, South African College School, City Mansions, Cape Town, South Africa;
Rev. David Ross Fotheringham, M.A., 13 Maitland Park Road, Hampstead, N.W.; and
Bernard Guy Harrison, The Grove, Freshford, Somerset,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Robert R. Agnew, M.A., Senior Science Master, Pupil Teachers' Institute, 15 Roxburgh Street, Kelvinside, Glasgow (proposed by S. M. B. Gemmill);
William Barnett, National Club, 1 Whitehall Gardens, S.W., and Laprida 1563, Rosario, Argentine Republic (proposed by W. Redfern Kelly);
Arthur James Coleman, Lieut. R.N.R., R.D., Port Officer and Marine Magistrate, Malacca, Straits Settlements, and Frithville, Queen's Road, Horley, Surrey (proposed by F. W. Dyson);

Thomas Francis Connolly, M.Sc., Solar Physics Observatory, South Kensington, Kent House, Peterborough Road, Parson's Green, S.W. (proposed by W. J. S. Lockyer);

Rev. Edward Bruce Kirk, David Elder Lecturer on Astronomy, Glasgow Technical College, Barrhead, Scotland (proposed by George Forbes);

Alexander D. Ross, M.A., B.Sc., F.R.S.E., Lecturer on Natural Philosophy in the University of Glasgow, 7 Queen's Terrace, Glasgow (proposed by F. W. Dyson).

Mr. R. Inwards, Mr. F. W. Levander, and Mr. W. E. Rolston were appointed Auditors of the Treasurer's Accounts for 1910.

Sixty presents were announced as having been received since the last meeting, including, amongst others:—

A table of the Sun's rising and setting for the latitude of Barrow, written by John Harrison (autograph), presented by Mrs. Cross.

25 Charts of the Astrographic Chart of the Heavens, presented by the Tacubaya Observatory.

On the Accuracy of the Positions of the Star Images in the "Harvard Sky." By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Some years ago I convinced myself by a number of measures on some of the plates taken at Harvard Observatory, kindly furnished by Professor E. C. Pickering for the purpose, that the optical distortion was small over a large angular field, but I had not time to make a very complete investigation. Recently a number of measures of another plate have been made by Mr. G. H. Hamilton, a volunteer worker at the University Observatory, and he also compared the measures with computed positions. His measures and computations form the basis of the present note.

2. The chief result is that the optical distortion can be represented within the limits of attainable accuracy as a radial distortion

$$T = 10380'' \tan^3 \theta,$$

where θ is the angle from the optical centre. Thus when

$\theta = 10^\circ$,	$\tan^3 \theta = \cdot00548$,	$T = 57''$
$\theta = 15^\circ$,	$\tan^3 \theta = \cdot01925$,	$T = 200''$.

To those accustomed to measures on the Astrographic Chart these may seem large quantities, but it is to be remembered—

(*a*) That the Harvard Sky is a small scale chart, on a scale about $\frac{1}{11}$th of that of the Astrographic Chart. In linear measure the displacement at 10° is about $\frac{1}{60}$th of a réseau interval of 5^{mm}, and would be represented by ·017 ; that at 15° by ·060.

(*b*) The size of this distortion is not so important as the accuracy with which it follows any specified and simple law. If it can be easily allowed for, it is not a serious inconvenience.

3. Before coming to the measures themselves, it will be a convenience to consider what will be the effect of a distortion of this kind on rectangular coordinates. If we take the origin at the optical centre, the displacements in x and y will be

$$\Delta x = kr^3x/r \qquad \text{and} \qquad \Delta y = kr^3y/r$$

or

$$\Delta x = kx(x^2+y^2) \qquad \text{and} \qquad \Delta y = ky(x^2+y^2).$$

But we do not know where the optical centre is on the plate. We may hope that it is near the centre of the plate itself, but there will almost certainly be some error in this supposition, and we may suppose that the centre is really at a point (α, β). Hence the above expression for Δx must be replaced by

$$\begin{aligned}
\Delta x &= k(x-\alpha)\{(x-\alpha)^2+(y-\beta)^2\} \\
&= kx(x^2+y^2) \quad . \quad . \quad . \quad \text{(I)} \\
&\quad - 2k\alpha x^2 - 2k\beta xy \quad . \quad . \quad \text{(II)} \\
&\quad - k\alpha(x^2+y^2) \quad . \quad . \quad . \quad \text{(III)} \\
&\quad + k(3\alpha^2+\beta^2)x + 2k\alpha\beta y \quad . \quad \text{(IV)} \\
&\quad - k\alpha(\alpha^2+\beta^2) \quad . \quad . \quad . \quad \text{(V)}.
\end{aligned}$$

4. The term marked (I) gives the optical distortion as though we had chosen the correct optical centre.

The terms marked (II), with the corresponding terms in Δy, will be recognised as similar in form to the effect of "tilt"; *i.e.* of the plate being imperfectly normal to the line joining the centre of the O.G. to the assumed centre. To this extent the second order terms in the two cases are indistinguishable.

But the terms marked (III) give us the means of recognising a displacement of the optical centre apart from unknown "tilt."

For instance, when $x=0$, Δx contains the term $-k\alpha y^2$, which cannot arise from any form of tilt. We can therefore determine α and β separately from the Δx residuals when $y=0$ and the Δy residuals when $x=0$. Hence it is convenient to measure stars near the rectangular axes through the adopted plate centre.

Refraction.

5. On these large plates the correction for refraction must be specially investigated. Formulæ for computing the refraction were given in *Monthly Notices*, lvii. p. 133, but there were some unfortunate mistakes in the paper. In *Monthly Notices*, lxiii.

p. 139, Mr. Hinks has given the corrected expression as far as terms of the second order, viz.—

$$\Delta x/\beta = X - x(1+X^2) - yXY + x^2X(2+X^2) + xyY(1+2X^2) + y^2X(1+Y^2)$$
$$\Delta y/\beta = Y - xXY - y(1+Y^2) + x^2Y(1+X^2) + xyX(1+2Y^2) + y^2Y(2+Y^2),$$

x and y being the coordinates of the star, and X, Y of the zenith on the plane of the plate, and β the coefficient of refraction, all expressed in circular measure. Suppose the plate is taken at 45° zenith distance on the meridian. Then $X=0$, $Y=1$, and these expressions become

$$\Delta x_2/\beta = -x+xy$$
$$\Delta y_2/\beta = 1-2y+x^2+3y^2.$$

6. The values of x and y may be as large as tan 15° or 0·27: hence $3y^2$ is comparable with y, or the second order terms are comparable with the first. Thus the question of convergence arises, and we must go back to the original expression expanded in series, viz.:—

$$t = \beta \frac{1+x^2+y^2}{1+xX+yY+\beta\{x(x-X)+y(y-Y)\}}.$$

The value of β being 57″ or 1/3600, the squares of β may be neglected, and we are concerned simply with $(1+xX+yY)^{-1}$ When $X=0$ and $Y=1$ this becomes $(1+y)^{-1}$; and since y cannot exceed 0·3, the convergence is assured, though it may not be rapid. Possibly it may be most convenient to tabulate the above expression as it stands in a table of double entry in x and y. But for our present purpose we want the expansion in series, so as to see how far the cube terms might be confused with the optical distortion.

7. The terms of the third order are—

$$\Delta x_3/\beta = -\{(1+X^2)x+XYy\}\{(x^2+y^2)+(xX+yY)^2\}$$
$$\Delta y_3/\beta = -\{XYx+(1+Y^2)y\}\{(x^2+y^2)+(xX+yY)^2\}$$

containing the terms of the first order as factors. This occurs with all subsequent orders which are formed as follows:—

$$\Delta x_4/\beta = +\{(1+X^2)x+XYy\}\{(x^2+y^2)+(xX+yY)^2\}(xX+yY)$$
$$\Delta x_5/\beta = -\{(1+X^2)x+XYy\}\{(x^2+y^2)+(xX+yY)^2\}(xX+yY)^2$$

and so on, the successive orders decreasing successively in the ratio $(xX+yY)$.

Note that the terms of odd order are all negative. If we remember that refraction contracts the field to the first order, we can infer that all the odd powers contract the field (while the even powers displace the centre relatively to the edges in a manner not so readily formulated). The optical distortion on the Harvard

plates expands the field, and its effect is thus partly counteracted by refraction.

8. The particular plate we are going to study (H.S. 9, $21^h\ 0^m + 60°$) was taken on 1903 May 13 at $19^h\ 25^m$ G.M.T., with an hour's exposure. Assuming that the time $19^h\ 25^m$ corresponds to the middle of the exposure, the hour angle was 44° E., and we find

$$X = -\cdot599 \qquad Y = -\cdot053$$
$$X^2 = +\cdot358 \qquad XY = +\cdot031 \qquad Y^2 = +\cdot003.$$

The first order terms in refraction are thus, in the units adopted in the sequel,

$$\Delta\xi_1 = -1''\cdot19\xi - 0''\cdot03\eta, \qquad \Delta\eta_1 = -0''\cdot03\xi - 0''\cdot88\eta,$$

ξ and η being expressed in réseau intervals on the right, so that $\Delta\xi_1$ and $\Delta\eta_1$ are given in seconds of arc. [These units are not strictly consistent, but it is convenient to show the quantities $\Delta\xi$ and $\Delta\eta$ in a familiar unit, and to use the observed values of ξ, η in computation.] The quantities ξ and η vary from about -16 to $+16$; so that the term $-1''\cdot19\xi$ has values differing by nearly 40″ on opposite sides of the plate.

9. The second order terms are similarly

$$\Delta\xi_2 = -0''\cdot020\xi^2 - 0''\cdot001\xi\eta - 0''\cdot008\eta^2$$
$$\Delta\eta_2 = -0''\cdot001\xi^2 - 0''\cdot009\xi\eta - 0''\cdot001\eta^2.$$

The largest term here, $0''\cdot020\xi^2$, becomes 5″ at $\xi = \pm 16$. This is a serious quantity in many investigations, but is near the limit of what can be detected when $1^{mm} = 11'$, since it is represented by 0·001 of a réseau interval.

10. The third order terms are the first multiplied by the factor

$$\{\xi^2 + \eta^2 + (0\cdot60\ \xi + 0\cdot05\ \eta)^2\} \times 0\cdot00023,$$

the numerical factor being that for converting measures made in réseau intervals into circular measure. When $\xi = 16$, $\eta = 0$, this expression has the value 0·08; and since the first order terms have the value 20″, the third order have values comparable with 2″. This is not altogether negligible, but the optical distortion which is the main subject of investigation in what follows amounts in the same locality on the plate to 144″, and is thus of a totally different order. When $\eta = 0$ the optical distortion is represented by $+0''\cdot036\xi^3$, and the third order refraction term by $-0''\cdot0004\xi^3$. We may thus ignore refraction for the present.

11. The plate selected by Mr. Hamilton was Harvard Sky No. 9, centre at $21^h + 60°$, and covering a field of about 30° square. A print from a large réseau by Gautier, with 32 spaces of 5^{mm} each way (presented to the University Observatory by Mr. Saunder), was securely bound up with the plate, film to film. Stars were measured between the lines 16 and 17 in each coordinate. From one or two easily identified stars it was inferred that to

make the adopted centre ($x = 16$, $\eta = 16$) coincide with the point ($21^h\ 0^m\ +60°$) for 1875·0 it was necessary to apply the approximate corrections

$$\Delta x = -0'.7 \qquad \Delta y = +0'.2,$$

the unit being a réseau interval; and further to orient it for 1875·0, the corrections

$$\Delta x = +0'.04y \qquad \Delta y = -0'.04x.$$

The epoch 1875·0 was chosen so that the places of the catalogues of the *Astronomische Gesellschaft* might be directly available without applying precessions; but only a small part of this orientation correction is due to precession for 25 years, which has a factor ·005. Denoting the measures by x and y we thus have

$$\xi = x + 0'.04y - 16'.700, \qquad \eta = y - 0'.04x - 15'.800.$$

12. The approximate scale value was then deduced from a few known stars, and the R.A.'s and Decls. for 1875·0 were then computed from the values of ξ and η by the formulæ—

$$k \tan (q - D) = \eta, \qquad k \tan (\alpha - A) = \xi \sec q \cos (q - D),$$
$$\tan \delta = \tan q \cos (\alpha - A),$$

where

$$\log k = 1'.81994, \qquad D = 60°, \qquad A = 21^h\ 0^m\ 0^s.$$

This value of k corresponds to a value of about $52'$ for a réseau interval of 5^{mm} at the plate centre, or a focal length of 330^{mm}.

13. With these values of R.A. and Dec. the stars were found in the catalogues and the differences O − C formed for α and δ. To convert $\Delta\alpha$ and $\Delta\delta$ into $\Delta\xi$ and $\Delta\eta$ requires a little arithmetic, and it would have perhaps saved some trouble to start with the places in the catalogue and convert them into ξ and η, when $\Delta\xi$ and $\Delta\eta$ could have been formed direct. But it was not easy to select stars which would fall near the lines to be measured, for in using such large fields the curvature of the declination circles and the slope of the meridians become serious, and Mr. Hamilton found it simpler to work back from the measures, at any rate in the first instance.

14. But it becomes necessary to obtain formulæ for $\Delta\xi$ and $\Delta\eta$ in terms of $\Delta\alpha$ and $\Delta\delta$. The following are general:—

$$\Delta\eta = \sec (\alpha - A) \sec^2 (q - D) \cos^2 q \{\sec^2 \delta\Delta\delta + \tan \delta \tan (\alpha - A)\Delta\alpha\}$$
$$\Delta\xi = \cos q \sec (q - D) \sec^2 (\alpha - A)\, \Delta\alpha. \quad - \Delta\eta \tan (\alpha - A) \sin D.$$

Let us write

$$\Delta\xi = P\Delta\alpha + Q\Delta\delta$$
$$\Delta\eta = R\Delta\alpha + S\Delta\delta.$$

Then the important cases for us are—

When $\xi = 0$.

Then $\alpha = A$, $q = \delta$, $\eta = \tan(\delta - D)$; and

$P = \cos\delta \sec(\delta - D)$, $\quad Q = 0$

$R = 0$ $\quad S = \sec^2(\delta - D)$.

When $\eta = 0$.

Then $q = D$; $\xi = \tan(\alpha - A)\cos D$; $\tan\delta = \cos(\alpha - A)\tan D$.

$P = \sin^2 D \cos D \operatorname{cosec}^2\delta$, $Q = -\sin D \sin 2D \operatorname{cosec} 2\delta . \tan(\alpha - A)$.

$R = \sin D \cos D \tan(\alpha - A)$, $\quad S = \sin 2D \operatorname{cosec} 2\delta$.

[Note as a useful check that

$$\frac{P\Delta\alpha Q\Delta\delta}{R\Delta\alpha S\Delta\delta} = -\frac{\sin^2 D}{\sin^2\delta}.]$$

15. We will now take these cases in order for the particular plate. Of course the conditions $\xi = 0$ and $\eta = 0$ are only fulfilled approximately. To eliminate accidental errors and save arithmetic the stars were combined in groups of 3 or 4, and in what follows the figures refer to the means of such groups.

TABLE I.

Case $\xi = 0$. *Values of* $\Delta\xi$.

δ.	$\Delta\xi$.	y.	$358'' + 18''{\cdot}3Y$.	$O - C_1$.	$+7'' - 0''{\cdot}036Y^2$.	$O - C_2$.
46° 30′	91″	0·524	+ 93″	− 2″	0″	− 2″
47 26	112	1·654	113	− 1	+ 1	− 2
48 38	142	3·122	141	+ 1	+ 2	− 1
50 6	175	4·885	174	+ 1	+ 3	− 2
51 56	219	7·050	213	+ 6	+ 5	+ 1
52 47	238	8·050	231	+ 7	+ 5	+ 2
53 31	250	8·919	247	+ 3	+ 6	− 3
54 25	269	9·959	266	+ 3	+ 6	− 3
56 13	313	12·028	304	+ 9	+ 7	+ 2
57 46	346	13·831	337	+ 9	+ 7	+ 2
59 8	372	15·411	365	+ 7	+ 7	0
59 40	385	16·016	377	+ 8	+ 7	+ 1
60 7	395	16·557	387	+ 8	+ 7	+ 1
60 25	401	16·868	392	+ 9	+ 7	+ 2
61 33	423	18·195	416	+ 7	+ 7	0
62 28	439	19·292	437	+ 2	+ 6	− 4
63 6	455	19·975	449	+ 6	+ 6	0
64 36	487	21·715	481	+ 6	+ 5	+ 1
65 29	498	22·758	500	− 2	+ 5	− 7
66 7	512	23·487	513	− 1	+ 4	− 5
66 35	524	24·029	523	+ 1	+ 4	− 3
66 48	533	24·289	528	+ 5	+ 4	+ 1
67 50	558	25·513	550	+ 8	+ 3	+ 5
69 4	587	27·024	+578	+ 9	+ 2	+ 7

The declinations go beyond this on the plate, but no star catalogue was available at the time of the measures, though the catalogue for $70^\circ - 75^\circ$ was received in July.

16. The first column shows the mean declination of the group, required for conversion of $\Delta\alpha$ into $\Delta\xi$ by the formulæ of § 14,

$$\Delta\xi = \cos\delta \sec(\delta - D)\,\Delta\alpha.$$

$\Delta\xi$ thus calculated is shown in the next column. Then is given the mean measured y coordinate of the group. The computations were commenced when little was known as to the probable results, and provisional constants and factors were adopted which later experience would have modified. Thus it was estimated that the optical centre fell near $y = 15$, and accordingly $Y = y - 15$ was used as argument instead of $y - 16$, which would have been more convenient in some ways. But the correction can be made at the end. The chief correction is obviously a linear term in Y, shown as $358'' + 18''{\cdot}3Y$ in the 4th column. Guided by § 4 we are now to look for a term in Y^2. This is clearly very small. In the 5th column is shown the value of a term $-7'' + 0''{\cdot}036Y^2$, which is about right, and gives the residuals in the 7th column. Only two of these reach $7''$ ($= 0{\cdot}002$ of a réseau interval); and though there are still traces of systematic error, we need not at present trouble further about Table I. than to note that the a of § 4 is approximately one réseau interval: *i.e.* $a = -1$.

17. Coming next to the values of $\Delta\eta$ when $\xi = 0$ calculated from $\Delta\delta$ and shown in the first column of Table II., the important term $+0''{\cdot}036Y^3$ (where $Y = y - 15$, and was adopted for purposes of convenient computation merely) is shown in the second column, and the effect of its application in the third. In the fourth a linear term in y is applied, found by trial; then a linear term in x, adopted from Table IV.; these linear terms requiring no comment at present. In column 6 is shown a term in Y^2, and the residuals $O - C_2$ are found still to require a small linear correction in y. The final residuals $O - C_3$ are small, with the exception of the first and second.

[TABLE.

TABLE II.

Case $\xi = 0$. *Values of* $\Delta\eta$ *and Comparisons.*

$\Delta\eta$.	$C_1 = +0''\cdot036Y^3$.	$O - C_1$.	$+100'' + 5''y$.	Linear in x.	$C_2 = -0''\cdot14Y^2$.	$O - C_2$.	$O - C_3$.	$O - C_4$.	$C_3 - C_4$.
$-211''$	$-110''$	$-101''$	$+2''$	$-11''$	$-30''$	$+19''$	$+19''$	$+7''$	$-12''$
−206	− 85	−121	−13	−13	−25	+12	+12	+3	− 9
−193	− 61	−132	−16	−23	−20	− 3	− 2	−7	− 5
−168	− 36	−132	− 8	−15	−14	− 1	0	−2	− 2
−152	− 18	−134	+ 1	−11	− 9	− 2	0	+1	+ 1
−155	− 12	−143	− 3	−12	− 7	− 5	− 3	−4	− 1
−160	− 7	−153	− 8	−12	− 5	− 7	− 5	−4	+ 1
−153	− 5	−148	+ 2	− 3	− 4	+ 1	+ 3	+2	− 1
−151	− 1	−150	+10	− 1	− 1	0	+ 3	+3	0
−161	0	−161	+ 8	− 2	0	− 2	+ 1	+1	0
−176	0	−176	+ 1	− 5	0	− 5	− 1	−2	− 1
−167	0	−167	+13	+ 1	0	+ 1	+ 5	+4	− 1
−185	0	−185	− 3	− 8	0	− 8	− 4	−5	− 1
−166	0	−166	+18	+ 1	0	+ 1	+ 5	+3	− 2
−183	+ 1	−184	+ 7	− 4	− 1	− 3	+ 1	+1	0
−194	+ 2	−196	0	− 5	− 2	− 3	+ 1	+2	+ 1
−187	+ 4	−191	+ 9	− 1	− 3	+ 2	+ 7	+5	− 2
−197	+ 11	−208	0	− 8	− 6	− 2	+ 3	+4	+ 1
−210	+ 17	−227	−14	−17	− 8	− 9	− 4	−2	+ 2
−205	+ 22	−227	−10	−15	−10	− 5	+ 1	+ 2	+ 1
−205	+ 26	−231	−11	−17	−11	− 6	0	+ 1	+ 1
−206	+ 29	−235	−14	−21	−12	− 9	− 3	− 2	+ 1
−202	+ 42	−244	−16	−23	−16	− 7	− 1	0	+ 1
−194	+ 62	−256	−21	−29	−20	− 9	− 2	+ 3	+ 5

18. All these terms were deduced by trial and error, except that in the first instance the coefficient of the cube term was found as $0''\cdot030$. The value $0''\cdot036$ was adopted from the $\Delta\xi$ series in Table III., and was not found to suit the observations quite so well. To compare the two hypotheses, the residuals obtained with the value $+0''\cdot030$ (and modified linear terms) are shown in the column $O - C_4$. The differences $(O - C_4) - (O - C_3)$ are shown in the column $C_3 - C_4$, and are only sensible near the ends of the series. It will be seen, therefore, that the exact value of the coefficient of the cube term is subject to some uncertainty.

19. Coming now to the case $\eta = 0$: in Table III. the first column shows the mean measured x for the group, and it will be seen that the series is a little more extensive than for y. (See Table I., column 3.) The next three columns give the parts of $\Delta\xi$ depending on $\Delta\alpha$ and $\Delta\delta$ respectively, and then the complete value of

$\Delta\xi$. Next comes the computed $0''{\cdot}036X^3$ where $X = x - 16$, and then come terms of lower orders determined empirically, viz.—

$$+0''{\cdot}25X^2 - 4''{\cdot}75X + 400''.$$

The linear correction for the small value of y is adopted from Table I.

20. In Table IV. the values of $\Delta\eta$ are similarly calculated, and they are nearly satisfied by a simple linear term in X. There is, however, a small term $-0''{\cdot}05X^2$, from which, on referring to § 4, we find $\beta = +1{\cdot}3$.

TABLE III.

Case $\eta = 0$. *Values of* $\Delta\xi$ *and Comparison.*

Mean x.	P$\Delta\alpha$.	Q$\Delta\delta$.	$\Delta\xi$.	$+0''{\cdot}036$ X^3.	O − C_1.	$+0''{\cdot}25X^2$.	$400'' - 4''{\cdot}75X$.	O − C_2.	Linear in y.	R.
	"	"	"		"	"	"			
2·423	+337	+ 80	+417	− 90	+507	+46	465	− 4	+ 5	+ 1
6·053	+390	+ 34	+424	− 36	+460	+25	448	−13	+ 6	− 7
8·476	+416	+ 12	+428	− 16	+444	+14	436	− 6	+ 5	− 1
10·969	+422	− 1	+421	− 7	+428	+ 6	424	− 2	+ 7	+ 5
13·957	+416	− 5	+411	0	+411	+ 1	409	+ 1	+ 3	+ 4
16·404	+379	+ 1	+380	0	+380	0	398	−18	+14	− 4
18·266	+369	+ 17	+386	0	+386	+ 1	390	− 5	+ 1	− 4
20·170	+349	+ 31	+380	+ 2	+378	+ 4	380	− 6	+ 4	− 2
22·729	+324	+ 61	+385	+ 11	+374	+11	368	− 5	+ 4	− 1
24·021	+311	+ 82	+393	+ 18	+325	+16	362	− 3	+ 4	+ 1
25·021	+302	+ 99	+401	+ 29	+372	+20	357	− 5	0	− 5
26·284	+294	+120	+414	+ 40	+374	+27	352	− 5	+ 4	− 1
28·770	+275	+172	+447	+ 76	+371	+41	339	− 9	+ 6	− 3
31·614	+281	+239	+520	+137	+383	+61	326	− 4	− 2	− 6

Cube Terms.

21. We may therefore consider as established a radial optical distortion, displacing the images outwards from the centre $\alpha = -1$ $\beta = +1{\cdot}3$ by $0''{\cdot}036r^3$. The complete expression for its effect is

[TABLE.

TABLE IV.

Case $\eta = 0$. *Values of* $\Delta\eta$ *and Comparison.*

R$\Delta\alpha$.	S$\Delta\delta$.	$\Delta\eta$.	$-19''X$ $-170''$.	O − C_1.	$+5''(y-17)$.		$+6''$ $-0''\cdot05\,X^2$.	O − C_2.
−119	+213	+ 94	+ 89	+ 5	− 2	+ 3	− 3	+ 6″
−103	+126	+ 23	+ 19	+ 4	− 3	+ 1	+ 1	0
− 85	+ 58	− 27	− 27	0	− 2	− 2	+ 3	− 5
− 56	− 14	− 70	− 74	+ 4	− 3	+ 1	+ 5	− 4
− 34	− 70	−104	−112	+ 8	− 2	+ 6	+ 6	0
+ 4	−172	−168	−178	+10	− 5	+ 5	+ 6	− 1
+ 26	−241	−215	−228	+13	− 2	+11	+ 6	+ 5
+ 39	−283	−244	−249	+ 5	− 2	+ 3	+ 5	− 2
+ 57	−341	−284	−298	+14	− 2	+12	+ 4	+ 8
+ 65	−384	−319	−322	+ 3	− 2	+ 1	+ 3	− 2
+ 72	−406	−334	−341	+ 7	− 1	+ 6	+ 2	+ 4
+ 79	−434	−355	−366	+11	− 2	+ 9	+ 1	+ 8
+ 90	−501	−411	−412	+ 1	− 3	− 2	− 2	0
+100	−574	−474	−468	− 6	0	− 6	− 6	0

in $\Delta\xi$, $+0''\cdot036\xi[(\xi+1)^2+(\eta-1\cdot3)^2] = +0''\cdot036\xi[\xi^2+\eta^2]$
$+0''\cdot07\xi^2 - 0''\cdot09\xi\eta + 0''\cdot09\xi.$

in $\Delta\eta$, $+0''\cdot036\eta[(\xi+1)^2+(\eta-1\cdot3)^2] = +0''\cdot036\xi[\xi^2+\eta^2]$
$+0''\cdot07\xi\eta - 0''\cdot09\eta^2 + 0''\cdot09\eta.$

Square Terms.

22. In Table II. we found a term $-0''\cdot14Y^2$ in $\Delta\eta$. Here Y was taken for convenience of computation, viz.—

$$Y = y - 15 = \eta + 0\cdot04\xi + 1\cdot470 \text{ approx.} \quad \text{(See § 11.)}$$

Now Y was also used in the cube term, and part of the square term will be due to the difference of origin. But we can easily allow for this.

$+0''\cdot036Y^3 = 0''\cdot036(\eta + 0\cdot04\xi + 1\cdot470)^3$
$= 0''\cdot036\eta^3 + 0''\cdot16\eta^2 + 0''\cdot24\eta + 0''\cdot12$ when $\xi = 0$
$-0''\cdot140Y^2 = -0\cdot14\eta^2 - 0\cdot42\eta + 0\cdot22$
$-5''\cdot0Y = -5\cdot00\eta + \text{const.}$

Total Corr. applied $= 0''\cdot036\eta^3 + 0\cdot02\eta^2 - 5\cdot18\eta + \text{const.}$
Due to opt. dist. $= 0\cdot036\eta^3 - 0\cdot09\eta^2 + 0\cdot09\eta + \text{const.}$

Remainder $+0\cdot11\eta^2 - 5\cdot27\eta + \text{const.}$

The square term is assignable to refraction and tilt of the plate.

In § 9 we have seen that the refraction is $-0''\cdot01\eta^2$; hence we have $+0''\cdot12\eta^2$ for tilt, involving a corresponding term $+0''\cdot12\xi\eta$ in $\Delta\xi$.

In Table III. we have similarly

$$X = x - 16 = \xi - 0\cdot04\eta + 0\cdot0 \text{ approx.}$$

Hence when $\eta = 0$ the terms applied are

$$+0''\cdot036\xi^3 + 0''\cdot25\xi^2 - 4\cdot75\xi + \text{const.}$$

$$\text{and opt. dist.} = +0''\cdot036\xi^3 + 0''\cdot07\xi^2 - 0\cdot09\xi$$

$$\text{Thus in the remainder } +0''\cdot18\xi^2 - 4''\cdot84\xi$$

the square term must be due to tilt, and implies a corresponding term $+0''\cdot18\xi\eta$ in $\Delta\eta$.

23. Let us determine the angular magnitude of the tilt thus indicated. If the normal from the centre of the O.G. does not fall at $\xi = 0$, $\eta = 0$, but at a point (k, l), the complete relation is

$$\xi + \Delta\xi = \frac{\xi - k}{1 + k\xi + l\eta} \qquad \eta + \Delta\eta = \frac{\eta - l}{1 + k\xi + l\eta}$$

(see, for instance, *Mon. Not.*, lxi. p. 429), all the quantities being expressed in circular measure. Thus

$$\Delta\xi = -k\xi^2 - l\xi\eta + k^2\xi + kl\eta - k$$
$$\Delta\eta = -k\xi\eta - l\eta^2 + kl\xi + l^2\eta - l$$

The value of one réseau interval (in which we have measured ξ and η) is $52'$ or $\cdot015$ in circular measure, while $\Delta\xi$ and $\Delta\eta$ have been expressed in seconds of arc. Adjusting the units we find from the square terms that

$$k = -0\cdot0039 \qquad l = -0\cdot0026$$

in circular measure, or in réseau intervals

$$k = -0\cdot26 \qquad l = -0\cdot18.$$

The plate was thus carefully adjusted, the normal falling very near the centre. The linear terms are therefore, when converted into the units here used,

$$\Delta\xi = +0''\cdot04\xi + 0''\cdot03\eta$$
$$\Delta\eta = +0''\cdot03\xi + 0''\cdot02\eta.$$

The constant may be neglected.

Linear Terms.

24. In $\Delta\xi$ we have

from Table I.,	$+18''\cdot30y$ or $+18''\cdot30\eta + 0''\cdot73\xi$
and from § 22	$-4''\cdot84\xi$
Altogether . . .	$+18''\cdot30\eta - 4''\cdot11\xi$

In $\Delta\eta$ we have

from Table IV., . .	$-19''\cdot0(\xi-0\cdot04\eta)$
from § 22	$-5''\cdot27\eta$
Altogether . .	$-19''\cdot0\xi\ -4''\cdot51\eta$

25. The scale values from $\Delta\xi$ and $\Delta\eta$ are thus $-4''\cdot1$ and $-4''\cdot5$, which ought to agree. The difference $0''\cdot4$ would make a difference of $12''$ in 30 réseau intervals, a quantity which should be noticeable if it exists in full. There is also a discrepancy in orientation. But we must remember refraction and tilt. In § 8 we have found for the first order terms in refraction—

$$\Delta\xi = -1''\cdot19\xi - 0''\cdot03\eta$$
$$\Delta\eta = -0''\cdot03\xi - 0''\cdot88\eta.$$

These are the *effects* of refraction, which contracts the field, and are to be subtracted from the observed $\Delta\xi$ and $\Delta\eta$. The effects of tilt are similarly (see § 23),—

$$\Delta\xi = +0''\cdot04\xi + 0''\cdot03\eta$$
$$\Delta\eta = +0''\cdot03\xi + 0''\cdot02\eta.$$

Hence, after correcting for refraction and tilt, our observed quantities will be

$$\Delta\xi = -4''\cdot1\xi + 18''\cdot3\eta + 1''\cdot2\xi = -2''\cdot9\xi + 18''\cdot3\eta$$
$$\Delta\eta = -19''\cdot0\xi - 4''\cdot5\eta + 0''\cdot9\eta = -19''\cdot0\xi - 3''\cdot6\eta.$$

The discrepancy in scale value has been increased rather than diminished, and that in orientation has not been touched.

26. At this point it was remembered that the assumed relations $\xi=0$ and $\eta=0$ were not strictly correct. The true relations were $x=16\cdot5$ and $y=16\cdot5$, the measures having been made between lines 16 and 17 in each case. Hence, recurring to § 11,

when $x=16\cdot5$, $\xi=16\cdot5+0\cdot04(\eta+16)-16\cdot7=0\ 4+0\cdot04\eta$
and when $y=16\cdot5$ $\eta=16\cdot5-0\cdot04(\xi+16)-15\cdot8=0\cdot1-0\cdot04\xi$.

Using these in § 14, it was found that they led to the approximate expressions

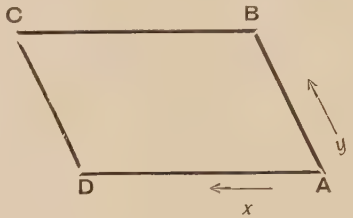
in	$\Delta\xi$	$+0''\cdot5\xi+0''\cdot3\eta$
and in	$\Delta\eta$	$-0''\cdot3\xi+1''\cdot0\eta$

so that we now have

$$\Delta\xi=(-2''\cdot9\xi+18''\cdot3\eta)+(0''\cdot5\xi+0''\cdot3\eta)=-2''\cdot4\xi+18''\cdot6\eta$$
$$\Delta\eta=-19''\cdot0\xi-3''\cdot6\eta-0''\cdot3\xi+1''\cdot0\eta=-19''\cdot3\xi-2''\cdot6\eta.$$

27. Considering the possibility of accidental error, the accordance in scale value may be considered satisfactory, but that in orientation is not quite. The difference of $0''\cdot7$ per réseau interval

of 52′ is about 1 in 4500 or ·00022. It would be explained if the réseau lines were inclined to one another at this angle, viz. 0°·011 or 0′·6. Now the errors of scale value and orientation of this particular réseau had been investigated by Mr. Bellamy, using the following simple method. He had photographed the réseau twice on the same plate, turning it round through 90° between the exposures.



It is easily seen that if AD exceeds AB in the ratio $1+k$ to 1, then we shall get scale values $1+k$ and $1-k$ when the two copies are compared. Again, if the angle BAD is acute, being θ defective from 90°, we shall get orientations $+\theta$ and $-\theta$ in the two co-ordinates. The measures showed that

$$k = +\cdot00012 \qquad \theta = +\cdot00012 = 28''$$

for the réseau generally. If, however, the middle lines (used by Mr. Hamilton for the selected stars here studied) be taken, the quantities become

$$k = +\cdot00014 \qquad \theta = +\cdot00023.$$

Substituting these later values in the equations as corrections depending on η in each case, they would be represented in our notation by $+0''\cdot4\eta$ in $\Delta\xi$ and $+0''\cdot7\eta$ in $\Delta\eta$; which now become

$$\Delta\xi = -\ 2''\cdot4\xi + 18''\cdot6\eta + 0''\cdot7\eta = -\ 2''\cdot4\xi + 19''\cdot3\eta$$
$$\Delta\eta = -19''\cdot3\xi -\ 2''\cdot6\eta + 0''\cdot4\eta = -19''\cdot3\xi -\ 2''\cdot2\eta.$$

We may fairly consider the outstanding scale-value difference of 0″·2 as accidental.

28. The adopted value of log k = 1·81994 is too large. To suit the plate as unaffected by refraction it should be reduced to 1·81864. Or as affected by refraction, about 1·81800.

Harvard Plate 25 ($6^h\ 0^m$, 0°).

29. We may now turn to another plate to see how far these results hold in other cases. Some years ago, soon after the receipt of the Harvard Sky, the plate ($6^h\ 0^m$, 0°) was bound up with a réseau and measures made near the two middle lines. This plate being taken at a greater zenith distance, we may first of all inquire whether the effect of refraction is likely to be more noticeable. The hour angle was 23° west, and we find X = +0·4, Y = +1·0. Thus the first order terms are (see § 7)

$$\Delta\xi_1 = -1''\cdot07\xi - 0''\cdot37\eta$$
$$\Delta\eta_1 = -0''\cdot37\xi - 1''\cdot83\eta.$$

The difference in orientation of $0''\cdot74$ in a réseau interval of $3100''$ is represented by a factor $\cdot00023$ and is by no means negligible. There is a nearly equal difference in scale value.

The second order terms are (see § 5)—

$$\Delta\xi_2 = +0''\cdot013\xi^2 + 0''\cdot018\xi\eta + 0''\cdot012\eta^2$$
$$\Delta\eta_2 = +0''\cdot017\xi^2 + 0''\cdot007\xi\eta + 0''\cdot644\eta^2.$$

And the third order terms are the first multiplied by the factor

$$\{\xi^2 + \eta^2 + (0\cdot4\xi + \eta)^2\} \times 0\cdot00023 \text{ (see § 10).}$$

The largest term is that of η^3, which is

$$-1''\cdot83 \times 2 \times \cdot00023 = 0''\cdot0009,$$

and, as before, this is small compared with the optical distortion.

The value adopted for log k was 1·81900.

30. Taking first the case $\xi = 0$, the values of η extended from $-15\cdot1$ to $+11\cdot5$. The cube term found empirically was $+\cdot0000075\eta^3$, or in our notation $0''\cdot024\eta^3$, as compared with $0''\cdot036\eta^3$ in the other plate. The difference $0''\cdot012\eta^3$ has the following values—

$\eta =$	0	5	10	11	12	13	14	15
$0''\cdot012\eta^3$	$0''$	$1''\cdot5$	$12''$	$16''$	$21''$	$27''$	$33''$	$41''$.

But, of course, this can be largely balanced by a linear term such as

$0''$	$7''\cdot5$	$15''$	$16''$	$18''$	$19''$	$21''$	$22''$,

the outstanding differences being

$0''$	$-6''$	$-3''$	$0''$	$+3''$	$+8''$	$+12''$	$+19''$,

or in réseau intervals

000	−002	−001	000	+001	+002	+004	+006.

The observed residuals are (with appropriate sign)

000	000	−002	+001	−002	+002	—	+006,

so that the results are fairly reconcilable. But the $+\cdot006$ depends chiefly on a large residual for a single star. The corrected log k now becomes 1·81802, in close agreement with that found from the former plate, as affected by refraction.

31. Coming now to the case $\eta = 0$, the residuals are sensibly satisfied with log $k = 1\cdot81900$ and a cube term $0''\cdot024\xi^3$. There is thus something to be said for taking the mean between the independent determinations, and putting

$$\text{distortion} = 0''\cdot030r^3$$
$$\text{with } \log k = 1''\cdot81850$$

as definitive values deduced from the mean of the two plates.

Summary.

§§ 3 and 4. Formulæ are derived for the effect of an optical distortion varying as the cube of the distance from a point near the plate centre; and it is shown how the coordinates of this point may be recognised.

§§ 5–10. The differential refraction is calculated to terms of high orders; but the third order terms are shown to be small compared with the optical distortion.

§§ 11–28. The measures of stars near two rectangular axes through the plate centre of Harvard Sky No. 9 are shown to be reconcilable with the star places if an optical (radial) distortion of $+0''\cdot036r^3$ be admitted from a point $(+1\cdot0, -1\cdot3)$, these coordinates and r being expressed in réseau intervals of about 52′ each. The value of this distortion at $r=20$ (in the plate corners) is thus 288″, or nearly 5′. The tilt of the plate is shown to be small; and the linear terms detected the slight systematic errors of inclination of the cross-lines and different scale value in the réseau used, which had been independently investigated by Mr. Bellamy.

§§ 29, 30. Measures on Harvard Sky No. 25 made some years ago confirm these results generally, but suggest a diminution of the coefficient of distortion from $0''\cdot036$ to $0''\cdot030$.

On Multiple Solutions in the Determination of Orbits from three Observations. By C. V. L. Charlier, Assoc. R.A.S.

Let ρ denote the distance of the planet (or comet) from the earth, r that from the sun. Put further

$$u=\frac{\rho}{R},\quad v=\frac{r}{R},$$

R being the radius vector of the earth; then u is known if v is given, and *vice versa*, as between these quantities there exists the relation

$$v^2=1+u^2+2nu\ . \qquad . \qquad . \qquad . \qquad (1)$$

where $n=-\cos\psi$ and ψ denotes the angle planet–earth–sun known from the observations.

Lagrange showed in the year 1778 that u (or v) can be determined from an equation of the form

$$u=\lambda\left(1-\frac{1}{v^3}\right), \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

λ denoting a constant, the value of which can be obtained from the observations. I call this equation the equation of Lagrange.

Using the relation (1) we obtain from (2) an equation of the eighth degree in u or in v, which in the last-named variable has the form

$$v^8 - v^6\{1 - n^2 + (n+\lambda)^2\} + 2\lambda(n+\lambda)v^3 - \lambda^2 = 0 \quad . \qquad (3)$$

It can be proved that $\lambda(n+\lambda)$ is always positive. On account of this theorem, and of the fact that the last term in (3) is always negative, there can be proved, with the help of the theorem of Cartesius, concerning the roots of the equation (3), the following propositions:—

The equation (3) has—

One root equal to unity. This root, corresponding to the orbit of the Earth, has nothing to do with our problem.

Two real positive roots.

One real negative root.

Four imaginary roots.

These propositions are generally valid.

The two positive roots alone need be considered. From (2) it follows that for λ positive we must have $v > 1$, for λ negative $v < 1$ (theorem of Lambert). Hence, if of the two positive roots one is < 1 and the other > 1, only one root is applicable. The solution is unequivocal. If the two positive roots are both > 1, or both < 1, there exist two solutions satisfying the problem.

From (3) we find the condition for the appearance of a double solution to be

$$1 - 3n\lambda > 0 \quad . \quad . \quad . \quad . \qquad (4)$$

The condition for a double solution has been first expressed by Laplace (in a somewhat different form) and most fully discussed by Oppolzer in his *Lehrbuch der Bahnbestimmung*. He has, however, failed to observe that the condition for the appearance of a double solution simply depends on the position of the planet (or comet) in space at the time of observation.

We have, indeed, according to (1) and (2)

$$n = \frac{v^2 - 1 - u^2}{2u},$$

$$\lambda = u/(1 - v^{-3}).$$

Substituting these values in (4), the condition for a double solution may be written

$$1 > \frac{v^2 - 1 - u^2}{2} \cdot \frac{3}{1 - 1/v^3} \quad . \quad . \quad . \qquad (4')$$

Through this inequality the space is divided into dominions with simple solutions and dominions with double solutions. These

different dominions are separated by the surfaces $u = 1$ (a sphere) and

$$u^2 = r^2 + \frac{2}{3r^3} - \frac{5}{3}$$

(the equation expressed in bipolar coordinates).

Evidently we have to do with surfaces of revolution with the radius Earth–Sun as axis of symmetry. The following figure gives a section of these surfaces.

When the planet (or comet) is in the dominion A or the dominion C we have double solutions of the problem of orbits. In B and D the solution is simple.

The comet 1910 *a*, regarding which Tscherny has shown that a multiple solution occurs, was at the time of its discovery in the dominion A.

Such double solutions are, however, by no means rare. This can be inferred directly from the figure, and is confirmed by a statistical research.

Regarding these points, and their consequences in the problem of orbits, I refer to the more circumstantial investigation in the *Meddelanden* from the Observatory of Lund.

1910 *November* 26.

Addendum.

In response to a suggestion from the Secretaries that it would be interesting to have the statistics referred to above, Professor Charlier has kindly written as follows, under date 1910 December 5 :—

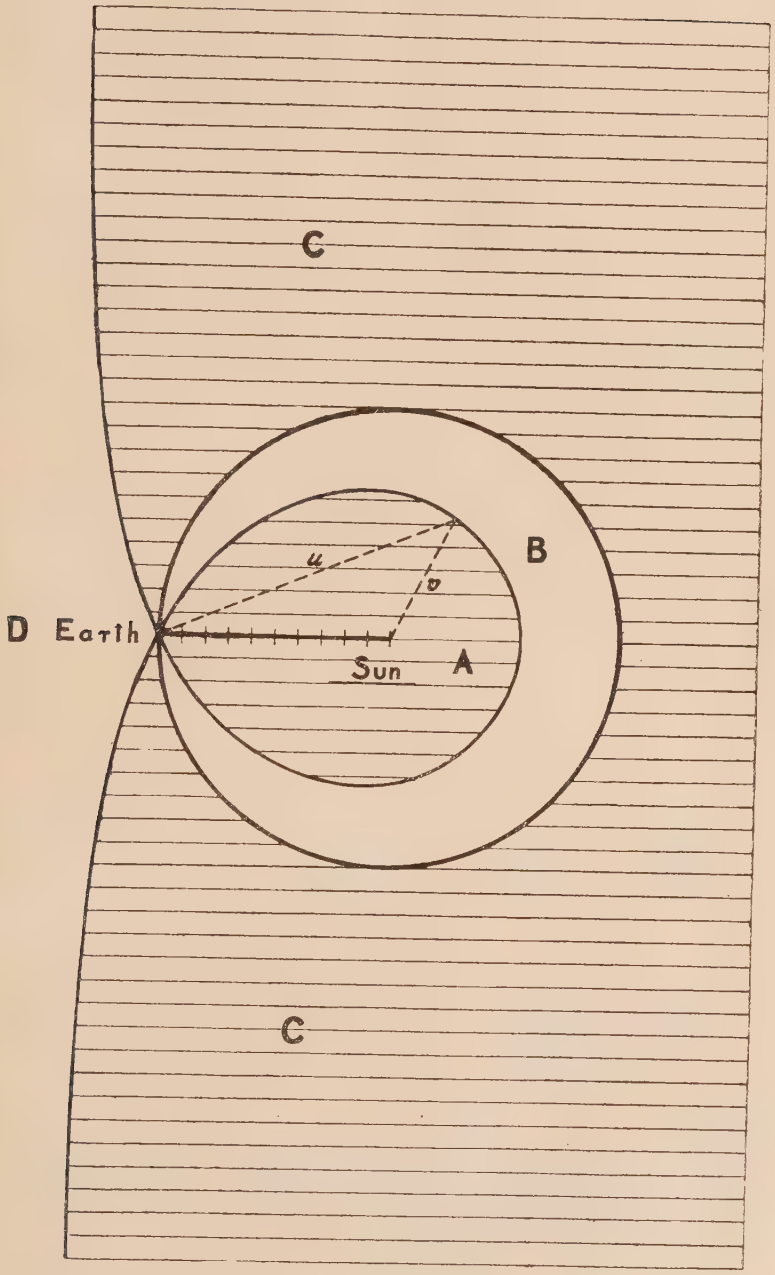
"I first examined the fourteen comets discovered in the last five years, and found the following numbers :—

"Region A	1 comet.	Region B	2 comets.
C	6	D	5
A+C	7	B+D	7

"Here double solutions are as frequent as single. At my request the assistant in the Observatory, Herr Åkesson, made a similar investigation of the comets in the years 1890–1900. The result was :—

"Region A	3 comets.	Region B	7 comets.
C	24	D	10
A+C	27	B+D	17

"Here there are twenty-seven cases of double solutions against seventeen single. You can infer from that that the method of Olbers for determining the orbit of a comet must be considered as rather unsatisfactory. Why the computer, even if a double solution is


C
B
u
v
D Earth
Sun
A
C

before him, is often led to the true orbit, is a question that deserves its own chapter. That, however, both solutions sometimes occur you may find, for instance, from the comets 1906 (Giacobini) and 1898 viii. (Chase), not to speak of the comet 1910*a*. The two former comets were, at their discovery, in the region C."

Note on an erroneous formula employed in the Tables of the Four Great Satellites of Jupiter. By Prof. R. A. Sampson, F.R.S.

The expressions which reduce the time of conjunction upon Jupiter's orbit to the point midway between immersion and emersion in eclipse are found as follows. If S, J, M are the centres of

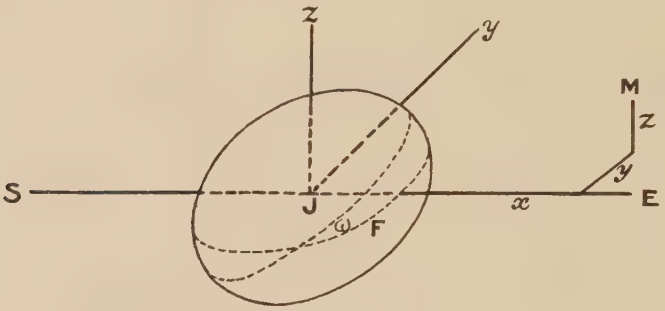


Fig. 1.

the Sun, Jupiter and the Satellite, ω the inclination of Jupiter's equator to his orbit, F the angle from the node of equator to SJ produced, $(1+\rho)R'$ the equatorial radius of Jupiter, D the distance SJ, then the coordinates of the satellite in eclipse satisfy the equation

$$0 = y^2 + z^2 + (2\rho + \rho^2)(z\cos\omega - y\sin\omega\cos F)^2 - (1+\rho)^2 R'^2 (1 + x/D)^2 \quad (1)$$

Cf. *Harvard Annals*, vol. lii., part ii., p. 228.

Or, suppressing small quantities with which we are not now concerned, and writing

$$y = r\Delta v, \quad z = r(s + s'\Delta v),$$

where Δv is angle measured from conjunction and projected upon Jupiter's orbit, s is the tangent of the latitude above the orbit at the moment of conjunction, and $s' = \frac{ds}{dv}$, we have the equation

$$A(\Delta v)^2 + 2H(\Delta v) + B = 0,$$

where

$$A = 1 + \ldots, \quad H = \{1 + 2\rho + \rho^2 + \ldots\} ss' - (2\rho + \rho^2)\, s \sin\omega \cos\omega \cos F$$

and the solution

$$\Delta v = -\frac{H}{A} \pm \sqrt{\left(\frac{H^2}{A^2} - \frac{B}{A}\right)}$$

gives us the so-called "Reduction to Middle" – H/A, or approximately $-(1+\rho)^2\, ss' + (2\rho + \rho^2)\, s \sin\omega \cos\omega \cos F$ . . . . (2) which must then be converted into time in proportion with the synodic motion.

This form applies for eclipses and, with certain additions, for occultations. The argument F is then equal to the satellite's principal argument of latitude, and the second term becomes confounded with a portion of the first, its chief effect being to reduce the first by about one-seventh part.

If now we turn from the theory of eclipses to shadows, when the satellite is in the neighbourhood of inferior conjunction, equation (1) still applies, and it seems at first sight evident that (2) is still the form of the Reduction, when s, s' both changing sign from the values they had at superior conjunction, the former term remains unaltered, but the latter, in place of diminish-

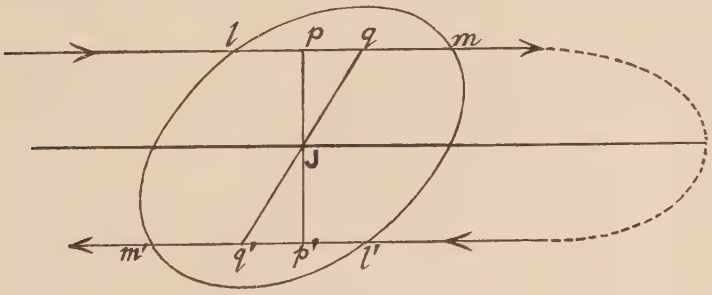


Fig. 2.

ing, now increases the value of the former by about one-seventh. Allowing for their completer expression, these are the forms I have used in my tables.

It is not easy to see that there is room for error in the above formulæ, and I had no suspicion of them until Dr. Cowell asked me for the reason of the difference from superior to inferior conjunction. In reply I quoted the formula (2), pointing to the term which changes sign; but, not dismissing the matter from mind, I looked further for a common-sense explanation. It then became clear that there must somewhere be an error, because there was no room for a difference of sign of the kind expected.

Looking towards S, let the planet's disc be as shown, and let lm be the path of the satellite in front of it at superior conjunc-

tion, $l'm'$ the path behind it at inferior conjunction, and consider the case when $s' = 0$ or the chords are parallel to Jupiter's orbit so that the first term of the Reduction is zero; then drawing Jq conjugate to lm, the length pq would measure the second term, which vanishes if the disc is a circle or if Jupiter's equator coincides with his orbit. Now it is evident that if pq is additive to the point of conjunction in the first case, $p'q'$ is equally additive in the second case, in spite of the latitude's change of sign, because the satellite is now moving in the opposite direction. Hence some error must lurk in the application of the formula (2) to the cases of shadow-transit and transit of the disc. It is the following.

Returning to the first figure, the satellite when approaching inferior conjunction is moving in the opposite direction to that at superior conjunction. Hence in place of $y = r\Delta v$, we should write $-y = r\Delta v$, while z retains its form unaltered, and this changes the sign of the second term in H, so that the form of Reduction appropriate to shadows and transits of the disc is not (2) but

$$-(1+\rho)^2 ss' - (2\rho+\rho^2)\, s \sin\omega \cos\omega \cos F \quad . \quad . \quad . \quad . \quad (3)$$

Remembering the value of F, the formulas (2), (3) when written out in terms of the arguments of the satellite's latitude assume identical forms.

There is a minor term of the semiduration arising from the same cause.

Probably the observations of shadows and transits, to which alone this error applies, would not have led one to remark the errors introduced, but that does not touch the question. It is an oversight of the most unfortunate character, inventing distinctions where none exist, causing the calculation and printing of several considerable tables which are superfluous, and increasing by 30 per cent. the leading term of the Reduction. It is true that the majority of the places where it declares itself could be set right by a very simple precept, directing the use for all phenomena of the tables which have been prepared for eclipses and occultations, but owing to details of the arrangement this would introduce other difficulties. After considering very carefully the possible courses, I have decided to reprint the whole of the Reductions for each satellite, and to issue them with an explanation as an erratum. In the form in which I am doing this, the tables will then be as straightforward to use as, and indeed a little simpler than, they are at present.

On the Mean or Perihelion Distances of Comets.

By R. T. A. Innes.

1.

It seems to be the invariable practice to make no direct determination of a planet's or comet's mean distance or perihelion distance, but to make use of the observed mean motion, which, combined with the Gaussian constant or some similar elliptic equation, will yield the value in terms of the Earth's mean distance. The weak point in this procedure is that observation does not furnish the mean elliptic motion (in the case of re-entering orbits), but the mean motion plus a virtually constant part which is due to the action of the planets. It is true that a certain part of the mean distance of any given body is also due to planetary action, but we cannot equate these two parts by means of the expression

$$a^3n^2 = k^2(1 + m).$$

Although this is well known, it has been generally assumed that the error introduced by its neglect is of no importance. Le Verrier, in his theories of the planets, assures us that this is so, and so it is except in the case of the theory of Jupiter and Saturn. In this case, through its neglect, Le Verrier's theory of those two planets is unsound, and the resulting perturbations are wrong in the fourth significant figure. As I am dealing with this in another paper, further reference is omitted here. In the case of the Galilean satellites of Jupiter, the effect (which is increased by Jupiter's ellipticity) is duly allowed for by Laplace in his theory. The American astronomers Newcomb and Hill have included this effect throughout in their theories and tables of the major planets. If it is a case of computing the semi-axis major of a minor planet from the observed mean motion, this correction becomes considerable owing to the proximity of Jupiter: thus for Ceres it is

$$\delta \log a = -0{\cdot}0000154$$
$$\text{or} \quad \delta a \quad = -0{\cdot}000010.$$

As minor planets are generally observed near opposition, this error in the radius vector will have only a small effect; but if it is desired to find the mass of Jupiter by means of the perturbations of a minor planet, it will be necessary to use the correct value of a. References to the "books" hardly seem necessary, but Newcomb's remarks on p. 402 of vol. iii. of the *Astronomical Papers of the American Ephemeris* may be mentioned. In dealing with a comet moving in a parabola or in a large and elongated ellipse, the argument can be put in a different and simpler form than in the case of the planets.

2.

When a comet, moving in a parabolic orbit, is approaching the Sun, it is attracted not only by the Sun, but by the planets

as well, so that the relation between its distance from the Sun and its velocity will involve not $k^2(1+m)$ but $k^2(1+\Sigma m)$ in which Σm represents the sum of the masses of the comet and all the planets. The equations of motion,

$$\ddot{x}+k^2(1+m)\frac{x}{r^3}=k^2\Sigma m_1\left(\frac{x_1-x}{{\rho_1}^3}-\frac{x_1}{{r_1}^3}\right)$$

(with similar equations in y and z, which will also be omitted below) can be rearranged as follows :—

$$\ddot{x}+k^2(1+m)\frac{x}{r^3}+\Sigma k^2 m_1\frac{x}{{\rho_1}^3}=\Sigma k^2 m_1 x_1\left(\frac{1}{{\rho_1}^3}-\frac{1}{{r_1}^3}\right).$$

When the comet is very distant from the Sun, ρ_1 is closely equal to r, whilst in the course of a planet's revolution x_1 changes sign, and the sum of its values is nearly zero. With these approximations the equations of motion of the comet reduce to

$$\ddot{x}+k^2(1+\Sigma m)\frac{x}{r^3}=0 \qquad . \quad . \quad . \quad \text{(A)}$$

In the cases of comets $1+m$ may be taken equal to 1, but $1+\Sigma m$ is about 1·0012. Hence the perihelion distance found by the usual tables of parabolic motion will as often as not be in error in the ratio of 1 to $\sqrt[3]{1\cdot0012}$.

When the comet is close to the Sun the formulæ of (A) no longer hold, but there is not a compensation. Let us suppose that the comet is very close to the Sun and the planets considered are Jupiter and Saturn. Then r_1 is nearly equal to ρ_1, and the latter can be replaced by a_1, the planet's mean distance; thus (A) becomes

$$\ddot{x}+k^2\left(1+\Sigma m\frac{r^3}{{a_1}^3}\right)\frac{x}{r^3}=0.$$

These remarks simply restate for comets the fact that the equation of planetary elliptic motion

$$n^2a^3=k^2(1+m)$$

does not hold when there are more than two bodies concerned, so that, as with the planets, if the mean distance is deduced from its observed motion, a correction has to be applied. As this correction has been neglected in the cases of comets, it results that the velocity depends on one conic section and the radius vector on another. From the method of calculation followed in the finding of the elements, the heliocentric longitudes should not be greatly in error, but the radii-vectores will be wrong; hence the geocentric coordinates will all be in error by an amount depending on the relative positions of the Sun, Earth, and comet. An ephemeris correctly representing the real position of the comet is therefore impossible, and calculators, to get the best representation between observation and computation, are forced to fit elliptic or hyperbolic

elements to comets moving in parabolas, in which the excentricity will deviate from unity in the ratio of about 1 to 1·0004. This no doubt accounts for the inconclusive results of so many investigations of comets' orbits. These remarks, suitably modified, apply to comets moving in elliptical orbits whose semi-axes major are large. In the "Essay on the Return of Halley's Comet," by Drs. Cowell and Crommelin, to which the prize of the Astronomische Gesellschaft was awarded, the authors give the following two elements on p. 7 for 1835,

$$a = 17{\cdot}98871$$
$$n = 46''{\cdot}5057.$$

These numbers are related to each other by the elliptic theory, thus:—

$$\log k = 3{\cdot}5500066 \text{ (the Gaussian constant)}$$
$$\log n = 1{\cdot}6675062$$
$$1{\cdot}8825004 \times 2 \div 3 =$$
$$1{\cdot}2550003 = 17{\cdot}9887.$$

In the case of Halley's comet, I find that $\delta \log a = +0{\cdot}0001950$ or $a = 17{\cdot}9968$, a difference of 0·0081. Near perihelion this error will in great part be transferred to the excentricity, so that the actual effect during the period of the visibility of the comet will not be large. The same remark applies to the purely elliptic value of a on p. 11 of the same paper.

3.

The writer has not found any references to this subject in the classical treatises of Watson, Oppolzer, or Bauschinger. The only note come across is one by Mons. M. Louis Fabry in vol. xi. of the *Bulletin Astronomique*, entitled "Note sur le calcul du grand axe des orbites cometaires" (pp. 485–488), in which he derives the formula

$$\frac{1}{a} = \frac{1}{a_0} - \Sigma\frac{2m}{R},$$

and adds that the quantity $\Sigma 2m/R$ is of the same order of greatness as the values of $1/a$ obtained for some comets which have appeared to move on hyperbolas.

Mons. Fabry's concluding remark is that, even when a comet remains far from the great planets, the action of the latter is not negligible, and can in certain cases change an elliptical orbit of great aphelion distance into a slightly hyperbolic orbit. Mons. Fabry does not, however, refer to the inconsistency of using the observed motion, due to the action of the Sun and planets, in deriving the semi-axis major by a purely elliptical equation.

Summary.

(*a*) The elliptic equation between the mean motion and the semi-axis major does not hold in the solar system.

(*b*) In the cases of minor planets and with comets with large distances, the error in the mean distance (or perihelion distance of parabolic orbit) deduced from the elliptic formula will affect the 5th significant figure.

(*c*) Unless the Gaussian constant is suitably modified, it will be impossible to get a complete reconciliation between observation and theory.

Johannesburg:
6th June 1910.

Note on Mr. Innes' Paper. By A. C. D. Crommelin, D.Sc.

I have been given an opportunity of reading Mr. Innes' Paper in advance of publication, and am fully in accord with his proposition that in distant regions of space, outside the planetary system, the total attraction of the solar system is practically equivalent to that of an equal mass concentrated at the centre of gravity of the system, and that a body would move in a conic section about this point as focus. We (Mr. Cowell and myself) used this principle in our earlier treatment of the motion of Halley's comet (*Monthly Notices*, vol. lxvii., Nos. 6 and 8). In the outer half of the comet's orbit we considered the whole of the action of the terrestrial planets and the greater part of the action of Jupiter and Saturn as though they coalesced with the Sun, leaving only a small residual action of Jupiter and Saturn to be computed by mechanical quadratures. We claim no originality for this idea, and I mention it here merely to show that Mr Innes' principle is not new. When, however, he proceeds to extend it to the regions of space adjacent to or within the orbit of the Earth, I am unable to admit its applicability. The most important perturbing planet, Jupiter, is here at least four times as remote as the Sun, making the ratio of their attractions 1/16000, and the fraction is still further diminished by the fact that it is only the difference of action of Jupiter on the comet and the Sun that is effective in perturbing motion about the latter. Again, the perturbation may either increase or diminish the total attraction; so it is clearly wrong to systematically increase the Sun's attraction by 1 in 700, or in any other constant ratio.

Passing to the consideration of an osculating conic, it is clear that in deducing such an orbit only such forces can be considered as are capable of producing motion in a conic. The planetary actions in general are not so capable, though, when we are dealing with long periods of time, there is a small effect of this kind, which may be combined with the Sun's action; it is, however, a

much smaller fraction of his action than is stated by Mr. Innes; and further, whenever the perturbing planet is more remote from the Sun than the perturbed one, the result is a diminution, not an increase, of attraction.

It is interesting to give, in this connection, Professor Newcomb's results for the corrections to the logarithms of the mean distances of the four inner planets arising in this way. The logarithm is, in the first instance, computed by the formula $a^3n^2 = k^2(1+m)$, and the corrections, in the ninth decimal, arising from the different planets, are given in the following table:—

		Perturbed Planet.			
		☿	♀	⊕	♂
Perturbing Planet.	☿	...	+ 31	0	+ 24
	♀	− 40	...	+634	+ 431
	⊕	− 15	− 189	...	+ 681
	♂	0	− 3	− 12	...
	♃	− 29	− 190	− 512	− 1920
	♄	− 1	− 9	− 24	− 87
	♅	0	0	0	− 2
	Sum	− 85	− 360	+ 86	− 873

We see that the effect is at most 1 in the sixth decimal, and that it is not constant in sign. The action of a planet on one nearer the Sun is negative, as we might expect. It will be seen how rapidly the resultant effect varies when passing a planetary orbit. It is hardly necessary to point out that the above is not the whole action of each planet on the other, but only that small fraction of it that is capable of being combined with the Sun's to produce elliptical motion.

I pass on to consider Mr. Innes' criticisms on a few points in our "Essay on the Return of Halley's Comet" (*Publ. der Astr. Gesells.*, xxiii.). I may recall that in that essay it was necessary to make a preliminary calculation by the older methods in order to obtain the correct mean motion or semi-axis major at the 1835 perihelion. All the elements except this were taken from Westphalen's discussion, but he had not investigated a, but used Rosenberger's value, which could not be taken as definitive, since he never finished the discussion for the few months preceding perihelion. Our object in finding this preliminary orbit was to obtain rectangular coordinates of the comet on two dates, viz. Paris noon on 1835 Nov. 16 and 18, from which to build up its motion by quadratures. We never expected that the mean motion that we adopted would prove absolutely correct; we were quite satisfied if it was near enough to represent the date of perihelion in 1759 within a day or two, for we knew that this would suffice to give the planetary perturbations accurately, and that the computed date in 1910 deduced from our tabular orbit could be corrected by

applying a correction practically equal to the discordance between our tabular result and observation in 1759. Hence we deemed it quite near enough for our purpose to treat the comet during the interval between our initial dates as continuing to move in the osculating ellipse. We were aware that this ellipse is really changing continually, but the change in the two days was negligible for our purpose. The result amply justified our judgment in this respect, for our tabular orbit represents the time of perihelion in 1759 within $\frac{1}{2}$ day, and in 1910 within $2\frac{1}{2}$ days. On the other hand, if we adopted the value of a given by Mr. Innes, viz. 17·9968, instead of our 17·9887, it is easy to see that the tabular period corresponding to a would be increased by 18·8 days. It needs no further proof to show that his value of a is wrong, for he will not dispute that, once we started the mechanical quadratures, the whole of the planetary action has been applied, and the observations in 1759, 1910, vindicate our value of a.

To show how slight the error is of taking the osculating ellipse as continuing unaltered for the two days, I give the values of the planetary attractions compared with the Sun's attraction at the date of perihelion in 1835. The first line gives the whole attraction of the planets in the three coordinates (which is the quantity to use if we are considering motion about the centre of gravity of the solar system); the second line gives the differential attraction of the planets on the Sun and on the comet (which is the quantity to use if we are considering motion about the Sun).

	Sun's attraction divided by		
Whole attraction of planets	110,000	90,000	250,000
Differential ,, ,,	180,000	210,000	160,000
Coordinate	x	y	z

We did not imagine that these attractions were absolutely non-existent, but we estimated, and rightly, that they might well be neglected for the two days in our preliminary investigation, compared with the necessary uncertainty of our initial value of a; in our final orbit their effect is fully introduced.

Photographic and Visual Observations of Halley's Comet (1909 *c*), *Daniel's Comet* (1909 *e*), *and Comet* 1910 *a*, *made at the Radcliffe Observatory, Oxford.* By Dr. A. A. Rambaut, F.R.S., Radcliffe Observer. (Plate 5.)

In a paper entitled "Places of Halley's Comet, 1909–1910, deduced from photographs taken at the Radcliffe Observatory, Oxford," and published in the *Monthly Notices*, vol. lxx. 6, the results of observations of Halley's Comet made at this Observatory during the months of November and December 1909 and January and February 1910 have already appeared.

The present paper contains the results of observations (visual and photographic) of the same object after the date of perihelion passage, to which are added the results of observations of a similar character made on Daniel's Comet of last year and on the Great Comet of January last.

The observers are denoted throughout by our usual abbreviated symbols, as follows:—

A.R., Dr. A. A. Rambaut; W., Mr. W. Wickham; R., Mr. W. H. Robinson; H.B., Mr. H. G. S. Barrett; B., Mr. J. G. Balk. Mr. Gerrard Rambaut also has occasionally assisted me, as indicated by his initials G.R.

Photographic Observations.

The method of deducing the place of the comet from the photograph was in each case similar to that described in the paper referred to above.

The results obtained are exhibited in the following tables:—

[Table.

Places of Halley's Comet, deduced from Photographs taken after Perihelion Passage, 1910, at the Radcliffe Observatory, Oxford.

Date.	G.M.T.	Local Sidereal Time.	Apparent R.A. of Comet.	Corr. for Parallax in R.A. (p).	Log. $p\times\Delta$.	Apparent N.P.D. of Comet.	Corr. for Parallax in N.P.D. (q).	Log. $q\times\Delta$.	Observers.	Ref.
1910.	h m s	h m s	h m s	s		° ′ ″	″			
May 23	9 36 54	13 34 19	7 57 38·55	+1·79	9·5670	78 15 44·6	−32·1	0·8204	A.R. W.	(*a*)
23	9 54 18	13 51 46	7 58 2·23	+1·80	9·5690	78 17 25·2	−32·4	0·8257	A.R. W.	(*b*)
25	9 55 55	14 1 16	8 48 18·75	+1·34	9·5549	82 8 42·2	−24·8	0·8228	W. R.	(*c*)
31	9 47 14	14 16 13	9 50 32·78	+0·69	9·5229	87 30 43·9	−13·9	0·8303	A.R. H.B.	(*d*)
June 2	9 39 34	14 16 25	10 0 14·09	+0·58	9·5139	88 22 50·4	−12·1	0·8323	A.R. R.	(*e*)
2	10 7 48	14 44 43	10 0 19·06	+0·61	9·5359	88 23 17·5	−12·1	0·8333	A R. R.	(*f*)

No astrographic catalogue was available covering the region through which the comet passed on the above dates. The measures were therefore compared with standard co-ordinates computed for the selected stars, the spherical co-ordinates of which were brought up from the sources indicated in the following table.

Authority for Adopted Places of Comparison Stars.

(*a*) Leipzig I., *A. G. Catalogue*, 3196, 3198, 3225, 3245.
(*b*) ,, ,, 3225, 3238, 3251.
(*c*) Leipzig II., ,, 4850, 4851, 4859, 4866, 4876.
(*d*) Albany, ,, 3914, 3916, 3917, and Küstner (1900), 4381.
(*e*) (*f*) ,, ,, 3942, 3943, 3946, 3958.

[For Observers' Remarks, see Visual Observations of Halley's Comet.]

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·80, and the geocentric distances, Δ, are taken from the *Astronomische Nachrichten*, Nos. 4379 and 4411.

Places of Comet 1909 *e* (*Daniel*), *deduced from Photographs taken at the Radcliffe Observatory, Oxford.*

Date.	G.M.T.			Local Sidereal Time.			Apparent R.A. of Comet.			Corr. for Parallax. in R.A. (p).	Log. $p \times \Delta$.	Apparent N.P.D. of Comet.			Corr. for Parallax in N.P.D. (q).	Log. $q \times \Delta$.	Observers.	Ref.
1909.	h	m	s	h	m	s	h	m	s	s		°	′	″	″			
Dec. 18	12	4	57	5	47	43	6	18	7·19	−0·10	8·8135	46	44	48·8	−2·1	0·1144	R. H.B.	(*a*)
31	13	6	0	7	40	12	6	17	22·20	+0·30	9·3091	39	7	43·4	−0·6	9·5830	R.	(*b*)
1910.																		
Jan. 19	10	31	28	6	20	9	6	22	2·74	0·00	...	33	56	39·4	+0·8	9·8357	R. H.B.	(*c*)

The co-ordinates for the comparison stars on 1909 December 18 are taken from the Helsingfors zone of the *Astrographic Catalogue*, *tome* iii., *cliché* 269, Nos. 34, 50, 62, 63, 64, and 98. The places of the comparison stars adopted for the observations of 1909 December 31 are those of the Cambridge (Mass.) *A.G. Catalogue*, the stars being Nos. 2484, 2485, 2489, and 2491 of that catalogue; and for 1910 January 19, Nos. 4494, 4508, 4525, and 4527 of the Helsingfors-Gotha *A.G. Catalogue* were employed.

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·80, and the geocentric distances, Δ, are taken from the *Astronomische Nachrichten*, Nos. 4376 and 4379.

Observers' Remarks.

(*a*) Condensation, magnitude 12; visible also in the 3-inch finder (power 23) as a faint luminous haze, enveloping a number of faint stars.

(*b*) Condensation, magnitude 12 or 12·5.

(*c*) Magnitude 13. Extremely difficult, especially at first, during a close appulse to a group of stars. Sky moonlit. Exposure two hours.

Comet 1910 *a*.

This comet was first photographed at the Radcliffe Observatory on January 25, although on the 20th and 22nd visual observations were made, and approximate places deduced from the circle-readings. The results of these observations are given in another part of the paper under the heading "Visual Observations."

The comet rapidly diminished in brightness, and after February 11 it was situated too low in the sky for accurate photographic work.

The results obtained from the photographs are exhibited in the following table.

Places of Comet 1910 *a*, *deduced from Photographs taken at the Radcliffe Observatory, Oxford.*

Date.	G.M.T.			Local Sidereal Time.			Apparent R.A. of Comet.			Corr. for Parallax in R.A. (p).	Log. $p \times \Delta$.	Apparent N.P.D. of Comet.			Corr. for Parallax in N.P.D. (q).	Log. $q \times \Delta$.	Observers.		Ref.
1910.	h	m	s	h	m	s	h	m	s	s		°	′	″	″				
Jan. 25	5	47	11	1	58	44	21	16	36·91	+0·31	9·5352	93	34	25·2	− 6·3	0·8430	A.R.	H.B.	(*a*)
25	5	58	21	2	9	56	21	16	38·69	+0·32	9·5420	93	33	47·9	− 6·3	0·8420	A.R.	H.B.	(*b*)
29	6	9	9	2	36	32	21	30	31·81	+0·28	9·5479	89	11	34·0	− 5·5	0·8355	A.R.	H.B.	(*c*)
29	6	20	42	2	48	7	21	30	33·41	+0·28	9·5527	89	11	6·4	− 5·5	0·8357	A.R.	H.B.	(*d*)
30	6	15	58	2	47	19	21	33	14·72	+0·28	9·5514	88	21	29·4	5·3	0·8346	A.R.	G.R.	(*e*)
Feb. 3	6	24	41	3	11	50	21	42	17·77	+0·25	9·5576	85	40	5·4	− 4·8	0·8320	R.	B.	(*f*)
4	6	45	58	3	37	7	21	44	(16·)	+0·25	9·5614	85	6	(24·)	− 4·7	0·8342	R.		(*g*)
11	6	33	36	3	52	19	21	55	31·21	+0·22	9·5642	82	0	59·4	− 4·1	0·8318	R.	H.B.	(*h*)

The measures were compared with standard co-ordinates computed for selected stars, whose places were deduced from the following sources:—

Authority for Adopted Places of Comparison Stars.

(*a*) (*b*)	Strassburg *A.G. Catalogue*,		7440, 7464, 7468.
(*c*) (*d*)	Nicolajew	,,	5472, 5475, 5482, Albany, A.G., 7548.
(*e*)	Albany	,,	7565, 7567, 7575, Küstner (1900), 9589.
(*f*)	,,	,,	7600, 7605, 7610, 7611.
(*h*)	Leipzig II.	,,	11053, 11062, 11065, 11070, 11071.

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·80, and the geocentric distances, Δ, are taken from the *Astronomische Nachrichten*, Nos. 4386 and 4387.

Remarks.

(*a*) (*b*) (*c*) (*d*) Comet's photographed image very difficult to measure; diameter 50″ to 60″.
(*g*) Image diffused; altitude 5°; only two faint star-trails found on plate. Resulting position of Comet approximate only.
(*h*) Altitude 7°.

Visual Observations of Halley's Comet and Comet 1910 *a*.

Halley's Comet.

In consequence of the comet's proximity to the Sun, which rendered it a difficult object in these latitudes, observations were discontinued after the month of February. The comet was last seen here before perihelion on February 28, when it was just visible in the 18-inch in strong twilight. After perihelion it was first looked for, and seen, with the 10-inch Barclay in the early morning of April 25.

The positions of the comet, as deduced from visual observations, being obtained from circle-readings only (except on May 22, as indicated by an asterisk), cannot lay claim to any great accuracy. They are, however, given in the following table, and may perhaps prove not wholly without interest:—

[Table.

Visual Observations of Comet Halley after Perihelion Passage, made at the Radcliffe Observatory, Oxford.

Date.	G.M.T.			Appar. R.A. of Comet.			Appar. N.P.D. of Comet.			Instrument.	Observers.		Ref.
1910.	h	m	s	h	m	s	°	′	″				
Apr. 24	16	28	52	23	50	18	82	12		Barclay (10-inch)	R.		(*a*)
May 20	8	24	12	5	25	53	70	58		,,	W.	R.	(*b*)
20	8	51	27	5	27	12	70	58		18-inch	A.R.	G.R.	(*c*)
22	9	15	30	7	18	52	75	44		,,	A.R.	G.R.	(*d*)
22	9	32	15*	7	19	21·3	75	44	51	,,	A.R.	G.R.	(*e*)
22	9	52	50	7	20	0	75	47		Barclay (10-inch)	H.B.		(*f*)
23	9	14	34	7	57	4	78	13		18-inch	A.R.	W.	(*g*)

Observers' Remarks.

(*a*) Apr. 24. Observed in strong twilight, using powers 45, 90, and 100. Nucleus only visible; magnitude 4, but image dull and of sensible diameter, approximately 8″. Too faint for the finder (2·7 inch) after $16^h\ 30^m$ G.M.T.

Apr. 29. $14^h\ 45^m$–$15^h\ 30^m$ G.M.T. In the 10-inch the comet appears about 3rd magnitude. Roundish, diffused, nebulosity of 40″ diameter. Nucleus stellar at times. An extension suspected in P.A. 290°–300°, but observations very difficult in rapidly increasing daylight. Thin haze over sky near eastern horizon. (A.R. and G.R.)

(*b*) May 20. Earliest opportunity for observing the comet since its passage to east of Sun.

The comet was detected in the 10-inch as a very faint nebulosity on a bright background of sunlit sky. In the finder the image was just discernible as a minute dull white patch.

(*c*) May 20. Rather brighter than ζ Tauri, 3rd mag.

(*d*) May 22. An extension or elongation of the nucleus was noticed by Dr. Rambaut in the direction P.A. 260°. This observation was corroborated by Dr. Cowell, who visited the observatory this evening.

(*e*) May 22. Position deduced from two transits of comet and star (Berl. A, *A.G.*, 2772).

(*f*) May 22. From the nucleus of the comet a beam of light proceeds towards the Sun, but curves southwards.

(*g*) May 23. Nucleus elongated (3″ or 4″) in P.A. 225°. The coma extends in same direction like a fan of about 90° vertical angle to a length of 70″. (A.R.)

Nucleus seems forked at s.p. end. Coma extends for about 2′ in P.A. 45°. (W.)

May 23. $9^h\ 15^m$ G.M.T. With the 10-inch Barclay telescope the outer edge of the coma is very indistinct, whilst the inner edge near the nucleus is sharply defined. Extending from the nucleus in the direction of the Sun is a narrow fan-shaped beam which curves round on either side, towards its extremity into the coma. It is brighter and the curving is more evident on the south side. (H.B.)

[Two photographs taken on this date (May 23) confirm the above observations.]

May 24. $9^h\ 55^m$ G.M.T. Nucleus visible to the naked eye. Magnitude 3·8, and surrounded by nebulosity 5′ in diameter. Tail just discernible in twilight and moonlight, and could be traced to a distance of 8°. Inclination of tail to vertical, 55°, towards the south. (R.)

May 25. 10^h G.M.T. The tail can be traced with the naked eye to a distance of about 30° (= distance between β Ursæ Maj. and Polaris). Rather difficult, very diffuse, no definite boundary; practically straight throughout its entire length. (H.B.)

May 31. $10^h\ 15^m$ G.M.T. Tail just visible to naked eye in twilight. Angle with vertical, 60°–65°. Brightness of head 3rd magnitude. (R.)

May 31. $10^h\ 30^m$ G.M.T. The tail is straight, and can be traced to a distance of 18°, or as far as ϕ Leonis, involving p^1 and p^2 Leonis. It is relatively bright for the first five degrees, but rather narrow with diffuse margins. (H.B.)

June 2. 10^h G.M.T. In the 18-inch, the horn extending from the nucleus at P.A. 120° is more marked than it was on May 31; the nucleus and comet generally are fainter than on that date. (A.R.)

Tail still visible to naked eye, clearly seen for five degrees and suspected five degrees farther. Magnitude of nucleus about 3. (R.)

Comet 1910 *a*.

The telegram announcing the discovery of this object was received here about 6 p.m. on January 18, when the comet had already set, and clouds prevented observations on the next afternoon. The first opportunity of observing the comet occurred late on the 20th, when it presented a striking appearance in the western sky, as described under "Observers' Remarks."

The places given in the following table were deduced from observations taken mostly when comparison-stars were not available, and generally in strong twilight, or at a low altitude. Except in two cases, as indicated by asterisks, the positions depend on circle-readings only.

Visual Observations of Comet 1910 *a, made at the Radcliffe Observatory, Oxford.*

Date.	G.M.T.			Appar. R.A. of Comet.			Appar. N.P.D. of Comet.			Instrument.	Observers.		Ref.
1910.	h	m	s	h	m	s	°	′	″				
Jan. 20	5	30	52	20	42	55	103	50		18-inch	A.R.	H.B.	(*a*)
22	5	31	17	21	0	13	98	44		,,	A.R.	H.B.	(*b*)
22	5	34	30	21	0	7	98	45		Barclay (10-inch)	R.		(*c*)
25	6	9	0	21	16	45	93	32		18-inch	A.R.	H.B.	(*d*)
26	5	33	36	21	20	37	92	17		,,	A.R.	H.B.	(*e*)
26	5	35	46	21	20	43	92	17		Barclay (10-inch)	R.		
28	5	56	47	21	27	36	90	7		,,	R.		(*f*)
29	5	43	0	21	30	29	89	12		,,	R.		
29	6	23	53*	21	30	32·7	89	10	57	,,	R.		(*g*)
Feb. 4	5	49	23*	21	44	11·4	85	7		,,	H.B.		(*h*)
15	6	47	5	22	0	56	80	39		18-inch	R.	H.B.	(*i*)

Observers' Remarks.

Jan. 20. The comet was picked up with the naked eye at $5^h\ 5^m$ G.M.T. In the Marlborough telescope (3·2-inch, power 54) the nucleus had a remarkably well-defined disc within the crescent-formed coma, and closely resembled, both in colour and diameter, the planet Mars, which was observed later in the evening. The northern boundary of the tail was very distinct. (R.)

At $5^h\ 18^m$ G.M.T. the tail measured $2\frac{1}{2}°$ in length, and shone conspicuously white against the red glow near the horizon. (A.R. and R.)

The inclination of the tail to the vertical estimated as 40° towards south. The nucleus could be followed until $5^h\ 37^m$ G.M.T., or to within a minute or so of the horizon. (R.)

(*a*) Jan. 20. Altitude $1\frac{1}{2}°$ only; corrections applied for refraction are therefore doubtful.

(*b*) Jan. 22. Estimated diameter of nucleus, 6″.

(*c*) Jan. 22 Altitude 6°. Tail inclined to vertical 45° towards south.

(*d*) Jan. 25. Single observation.

(*e*) Jan. 26. Estimated diameter of nucleus, 5″. Sky too thick for photography.

(*f*) Jan. 28. Observed in breaks of cloud.

(*g*) Jan. 29. Position obtained from two transits of comet and star (Nicolajew *A.G.*, 5468).

Jan. 29. $6^h\ 50^m$ G.M.T. From the head to a distance of about 10° the tail was straight; it then curved

Comet 1910a as observed at the Radcliffe Observatory, Oxford, 1910 Jan. 30d. 6h. 50m. G.M.T.

From a drawing by H. G. S. Barrett.

sensibly towards the south, enveloping α Pegasi; here a division in the tail was clearly discernible, the lower portion, below α Pegasi, being the more luminous. The tail could be traced to a distance of 40°, or to about midway between α and γ Pegasi. The bifurcated form gave the impression that the tail had met with some resistance, causing a part to deviate at a sharp angle. Long after the comet's nucleus had set, the tail and the neighbouring zodiacal light remained visible. (R.)

Jan. 30. $6^h\ 15^m$ G.M.T. Nucleus mag. 4 to naked eye. (R.)

Jan. 30. $6^h\ 50^m$ G.M.T. The tail of the comet was seen distinctly separated from the zodiacal light, and could be traced as far as ρ and η Piscium (or more than 50°); near this it merged into the zodiacal band. The northern edge of the tail was sharp up to a point between α and β Pegasi, whilst the southern edge was less distinct and not so well-defined. Sky transparently clear. (H.B.)

Plate 5 is a reproduction of a drawing by Mr. H. G. S. Barrett, which represents the appearance of the comet and the zodiacal light, at Oxford, on January 30. The drawing is made on a gnomonic projection with centre at α Pegasi. The four stars forming the Great Square of Pegasus are conspicuous near the middle of the picture, whilst close to the horizon and vertically below the centre of the Square is the planet Venus (altitude $1\frac{1}{2}$°).

(*h*) Feb. 4. The comet's position depends on two ring-micrometer transits of comet and star, Albany. *A.G.*, 7614.

(*i*) Feb. 15. Very faint, in twilight; altitude only $4\frac{1}{2}$°.

Radcliffe Observatory, Oxford:
1910 *December* 1.

Observations of Stars occulted by the Moon during the Eclipse of 1910 November 16, at the Radcliffe Observatory, Oxford.

(*Communicated by Dr. A. A. Rambaut, F.R.S., Radcliffe Observer.*)

This paper contains the results of observations at the Radcliffe Observatory, Oxford, of stars occulted by the Moon during the eclipse of 1910 Nov. 16. The observations were made by Mr. H. G. S. Barrett, with the Barclay equatorial of 10 inches aperture, using a power of 90, except in one observation where 100 was employed.

The numbers given in the first column of the table below refer to M. Th. Wittram's list of selected stars in the circular issued by him on behalf of the Russian Astronomical Society.

An interesting feature in this eclipse, pointed out by M. Wittram in his introductory remarks, is that it took place in the same region of the sky as that of 1891 November 15.

In the circular referred to, the position-angles and times of occultation for Oxford were given in the case of four stars for which the disappearance or reappearance took place during the total phase. But this number was supplemented by a useful list, published by Mr. Stevens in the "Observatory" magazine for November 1910, of occultations taking place during the partial phases of the eclipse.

The results obtained are exhibited in the following table :—

No. in Pulkowa List (Wittram).	No. in B.D.	Mag. (B D.)	G.M.T. of Observation.			
			Disappearance.	Ref.	Reappearance.	Ref.
	°		h m s		h m s	
15	+18 484	6·5	10 39 51·2	(*a*)	...	...
24	+17 558	9·4	10 53 11·4	(*b*)	...	...
21	+18 487	8·3	11 10 43·5	(*c*)	11 59 55·2	(*e*)
31	+18 489	8·6	11 44 51·1	(*d*)	12 47 17·3	...
38	+18 492	8·3	12 22 33·5	(*f*)	13 16 11·4	...
37	+18 491	9·5	12 35 7·8	...	...	...
41	+18 494	8·3	12 56 58·5	(*g*)	...	...
43	+18 496	9·5	12 58 17·2	(*h*)	...	...

Mr. Barrett's Remarks.

(*a*) Disappeared at bright limb ; recorded time uncertain by 2ˢ.

(*b*) May be 1ˢ in error. Position-angle about 115°. Observed with power 100.

(*c*) Disappeared at bright limb on the margin of a depression some distance from the advancing shadow.

(*d*) Angle about 90°. Star advanced a very little within the limb before disappearing instantaneously.

(*e*) Reappeared instantaneously. Moon's limb rather bright but observation good.

(*f*) Instantaneous.
(*g*) Magnitude 8. P.A. about 125°.
(*h*) *n.f.* No. 41. Faded considerably for about half a second, and then disappeared suddenly.

Throughout the eclipse exceedingly good definition prevailed; the Moon was easily visible, and much detail could be plainly made out.

The edge of the shadow for about 2′ was always of a dark-grey or neutral shade, merging gradually into a reddish tint.

Radcliffe Observatory, Oxford:
1910 *December* 3.

Occultations of Stars observed at Cambridge during the Eclipse of the Moon, 1910 *November* 16.

(*Communicated by Sir Robert Ball, Director of the Observatory.*)

In accordance with the request of the Russische Astronomische Gesellschaft, occultations of stars selected from Professor Wittram's list were made at the Cambridge Observatory.

The observations were made by Mr. Hinks with the Sheepshanks equatorial, of aperture 12 inches and focal length 19·3 feet. The reduced times depend upon clock corrections determined by Mr. Hartley with the 8-inch meridian circle, taking the mean of eye and ear and of chronograph determinations.

No. in Wittram's List.	B.D.	Mag.	D or R.	G.M.T.	Notes.
				h m s	
15	+18° 484	6·5	D	10 41 36·9	Poor observation. Limb very unsteady.
21	18° 487	8·3	D	11 12 34·8	Limb unsteady.
31	18° 489	8·6	D	11 46 55·2	Instantaneous.
21	...	...	R	12 1 55·6	
37	18° 491	9·5	D	12 37 6·8	Hung on the limb for a moment.
31	...	...	R	12 49 15·4	Good.
41	18° 494	8·3	D	12 58 11·5	Hung on the limb.
43	18° 496	9·5	D	12 59 45·4	Hung a little.
44	18° 497	9·5	D	13 3 51·4	Uncertain. Lost as it came to limb close to bright cusp.
38	...	...	R	13 18 12·6	Good.
41	...	...		13 27 51·1	

1910 *December* 3.

On the Dark Band surrounding the Polar Caps of Mars.

By E. M. Antoniadi.

In 1899 the writer was led by his observations to doubt the reality of the band seen round the polar snows of *Mars*;* and, since then, Dr. Johnstone Stoney, F.R.S., independently reached the same conclusion.† Later, in 1909, the writer noticed that the band does not appear on Professor Lowell's photographs made in 1907,‡ and further called attention to the fact that its generally uniform breadth on the drawings gravely disobeys the laws of perspective.§ The absence of the marking in question on the 1907 prints was further doubtfully ascribed to the possible, though not probable, agency of photographic irradiation,‖ as on those photographs the cap was very bright, and protruding beyond the limb. But an examination of the Mount Wilson glass positives of 1909 showed the cap to have been no larger than was seen visually at the time, while its intensity was fully comparable, on the same photographs, with that of the "continental" regions of the planet. Facts like these confirmed the writer in his belief in the subjectivity of the dark band.¶

Now, on p. 652 of vol. lxx. of the *Monthly Notices*, Professor Lowell questions the statement that the cap does not appear brighter on photographs taken in America in 1909, and demonstrates that his own negatives, obtained in light of λ 5600, give, on the print, a bright cap. But the negatives to which the writer made allusion were not taken, like Professor Lowell's, in λ 5600, but with "a Wallace red screen, transmitting light of wave-length greater than λ 5800."** That on these photographs the cap is practically not brighter than the "continents" is proved by the fact that the phenomenon did not strike only the writer, but also others. Thus, at the December 1909 meeting of the British Astronomical Association, it was remarked that, on the same positives on glass, "one curious point was that the polar cap was very little brighter, if at all brighter, than the principal bright regions of the planet"; and that "the explanation of that was very simple. The photographs were taken through a screen which allowed only the orange-yellow light to pass, and consequently those districts in the planet which were of the same tint as the screen showed just as bright as the white polar cap."†† Such independent testimony goes far to controvert Professor Lowell's contention that the writer could have denied "facts in order to support a previously conceived theory."

* "The objectivity of the dark band surrounding these snows is doubtful" (Observation of 1899 February 20 in Flammarion, *Mars*, ii. p. 448).

† This is given on the evidence of a friend, as the writer did not see Dr. Stoney's paper.

‡ *Astron. Nachr.*, No. 4358.

§ *Ibid.*, No. 4376. ‖ *Ibid.*, No. 4358.

¶ *Ibid.*, No. 4427.

** *Monthly Notices*, vol. lxx. p. 175.

†† *Journal* B.A.A., vol. xx. pp. 122–123.

But a cap reduced to its visual size and to the brightness of the "continental" regions cannot swamp, through photographic irradiation, a contiguous dark border, were the latter to have an objective existence. Detail finer than any ever drawn prior to 1909 was revealed by the splendid Mount Wilson photographs of *Mars*; and it is significant that while the latter show Deltoton Sinus triple (as first seen a fortnight previously in France *), they do not countenance the reality of a dark polar band, although the latter is so striking a feature in the smallest telescope.

The Equatorial Current of Jupiter in 1880.

By A. Stanley Williams.

During the past twenty-two years, from 1888 up to the present time, the velocity of the great equatorial current of Jupiter has been practically uniform, apart from comparatively small and apparently temporary variations from year to year, whereas the earlier observations indicate that the rotation period of the current had been almost half a minute shorter only a few years previously.† Owing to this remarkable circumstance, it seemed very desirable that a fresh investigation should be made as to the actual rate of rotation of the current, based on the observations of as many separate spots as possible, and going back as far as practicable in point of time.

Previous to 1880 the observational material is too rough and scanty to permit us to hope for any success in an investigation of this kind. The striking appearance presented by the red spot, it is true, had attracted almost universal attention to the planet in the preceding year,‡ but it was only in 1880 that much considera-

* Letter of the writer to Professor Schiaparelli, dated 1909 September 21, and published in the *Rivista di Astronomia e Scienze Affini* for 1909.

† Most of the determinations of rotation period prior to 1888 that appear to be at all reliable have been brought together in the *Monthly Notices*, vol. lvi. p. 147. The results are mostly derived from single spots.

‡ The red spot appears to have been independently detected by quite a number of observers about this time, and the writer still recalls the delight with which he independently discovered it for himself when observing the planet on the night of 1879 August 12, with a 2¾-inch refractor. It was then described as "a portion of a belt," very plain, and of a beautiful light coppery colour, extremely vivid, and in fine contrast with the yellow colour of the body of the planet. At Marth's instigation, a few observations of the time when the spot was in mid-transit across the disc were recorded with this instrument in the same year, but it was not until 1880 that, having become the fortunate possessor of a 5¼-inch reflector, the writer was in a position to commence what might, perhaps, be called serious work. Thanks to the investigations of Mr. W. F. Denning (*M.N.* vol. lix. p. 574), we are now acquainted with the early history of the red spot with certainty as far back as the year 1831. The earliest records of the object as a *red* spot would still appear to be those obtained with the Rosse 6-ft. reflector in 1873 (*M.N.* vol. xxxiv. p. 239).

tion was given to the determination of the motions of the other surface markings of Jupiter, whilst for the purpose in view we require numerous and accurate observations of a number of markings distributed as far as possible all round the planet. The present paper contains the results of an investigation into the velocity of the equatorial current in 1880. Incidentally, some light will be thrown upon another question of importance in connection with the study of the planet, namely, as to the relative efficiency or accuracy of the methods made use of for determining longitudes on Jupiter. According to one method, the position of a spot is measured directly with a micrometer from the preceding and following limbs of the planet, whilst in another one the time when a marking is in mid-transit across the disc is noted by the eye. It is well known that widely divergent views have existed as to the relative efficiency of the two methods.

In 1880, and also in the preceding year 1879, the appearance of the equatorial region of Jupiter differed very considerably from that which it has presented at any time during the last thirty years. The south equatorial belt was then narrow and single instead of being broad and double, the southernmost component not being visible, and the well-known "hollow" or red spot being non-existent.* The north equatorial belt was also for the most part single and narrow, whilst the bright equatorial zone was unusually narrow. In it were a number of large bright patches or spots, usually much elongated in an east and west direction, and irregular both in outline and brightness. From these larger patches smaller, brighter, and more definite condensations or spots projected into or indented the dark south equatorial belt. Two or three of these smaller spots were remarkably brilliant and definite, so that they much resembled the bright disc of a satellite in transit when just entering upon or leaving the planet. These peculiarities are somewhat important, for the reason that some of the observations refer to the large bright patches, and others to the smaller disc-like spots, whilst it is doubtful sometimes what was the feature actually observed. Some of the largest apparent discordances are doubtless to be ascribed to this cause.

Two of the smaller disc-like spots were particularly brilliant and definite. They are numbered III. and IV. Spot IV. is specially interesting because it is the well-known, long-enduring, white equatorial spot which was so assiduously observed during four or five consecutive years, above all by Mr. Denning at Bristol. Many of the observations of these two spots were brought together in a paper by Marth published in the *Monthly Notices*, vol. xlii. p. 428, but numerous additional observations have been added from various sources. In particular, mention should be made of a fine series of micrometer observations by the late Professor G. W. Hough at the Dearborn Observatory. At the time when Marth published the paper above mentioned, it was

* The following shoulder of the "hollow" made its appearance, however, towards the latter part of the apparition of 1880–81.

doubtful whether there were really two separate spots here or only one, and the question was left open by him. It now appears to be practically certain, however, that there were really two quite separate spots. The question has been more fully discussed in the remarks under spot IV.

The following lists contain all the observations, both published and unpublished, of equatorial spots made during the opposition of 1880–1 that have come to the writer's knowledge. The different columns will explain themselves, but the longitudes in the third column relate to a particular system (System I. of Marth).* In this system the daily rate of rotation is $878^{\circ}\cdot46$, corresponding to a rotation period of $9^{h}\ 50^{m}\ 7^{s}\cdot42$. The following abbreviations are used in column 4 to indicate the different observers :—

B. = E. E. Barnard.	L. = O. Lohse.
C. = H. Corder.	M. = J. L. M'Cance.
D. = W. F. Denning.	N. = L. Niesten.
De. = F. C. Dennett.	S. = J. F. J. Schmidt.
G. = G. T. Gwilliam.	T. = L. Trouvelot.
H. = G. W. Hough.	W. = A. S. Williams.
K. = J. Kortazzi.	

A few words should be said as to the manner in which the rotation periods have been derived. If the observations were free from all errors and invariably referred to exactly the same feature, in itself of invariable aspect, all that would be necessary would be to compare the two end observations, without regard to what the object observed may have been doing in between. Unfortunately, however, in reality not only are the observations subject to various forms of error, but the objects observed are not invariable, neither can we be sure that the same reference point has always been observed, particularly so when the observations are made by several different observers, using telescopes of various sizes, under very different observing conditions. It is almost always necessary, therefore, in practice, to consider more or less the whole mass of observations. In cases where there have been several apparently satisfactory observations near the commencement and end respectively of a series of observations, it has been the writer's practice to derive the rotation period from these observations alone. But the position is far more difficult when there are only one or two indifferent observations at the commencement or end of the series,† and the intermediate observations indicate a motion differing widely from that which would result from comparison of the end observations. In such cases it is necessary to base the result largely upon the whole mass of observations, and a good deal must necessarily be left to the judgment of the computer. Not-

* *M.N.*, vol. xlii. p. 427.

† Observations made near the commencement or end of a series are, unfortunately, frequently of poorer quality than usual, owing to the planet's position being then unfavourable for observation.

withstanding this, where there are a number of spots available, the mean results obtained by different computers seldom differ by any large quantity.*

White Spot I.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Aug. 28	14 14·5	1·5	T.	Nov. 2	13 11·9	349·3	H.
Oct. 18	9 30	356·0	W.	Dec. 30	7 56	343·6	M.

R = 9^h 50^m 1^s·60 (302 rotations). The spot was also seen and drawn by W. on 1880 October 28, but, owing to the small number of observations and the large intervening gaps, there is a little uncertainty whether the observations really all relate to the same spot.

White Spot II.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Nov. 2	13 53.0	14·4	H.	Nov. 25	12 40·5	14·8	T.
4	5 13	15·7	D.	26	8 25	17·5	D.
8	17 25·7	15·2	H.	29	5 4	10·1	D.
17	7 56	13·9	D.	1881. Feb. 23	[7 5]	[15·6]	D.

R = 9^h 50^m 6^s·73 (65 rotations). Owing to the large intervening gap, it is doubtful if the last observation really relates to the same spot as the others. It has therefore not been used in deriving the rotation period.

White Spot III.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
July 8	19 22·5	25·6	H.	Sept. 3	18 9·3	16·2	H.
9	14 57·4	22·2	L.	5	9 26·1	14·2	K.
16	14 9·7	22·6	L.	7	10 35·2	13·6	K.
17	19 43·9	24·7	H.	10	17 17·5	14·7	H.
24	15 54·7	24·1	H.	16	10 52·6	11·4	K.
Aug. 4	20 23·9	21·8	H.	16	11 2	17·1	W.
6	21 30·7	19·6	H.	18	11 50	3·7	D.
11	19 32·6	20·2	H.	21	8 50·2	9·8	K.
27	9 10·8	17·7	K.	24	15 33·4	11·4	H.
31	11 37·5±	21·4±	W.	Oct. 1	14 43·9	11·4	H.

* The following is a good illustration of this. From his observations of 23 equatorial spots in 1898, a rotation period of 9^h 50^m 23^s·6 was derived by Denning, whilst Phillips made it 9^h 50^m 24^s·2 from 19 spots observed by himself. Both these observers communicated their observations to the writer, who, combining them with a series of his own observations, found a rotation period of 9^h 50^m 24^s·0 from 38 spots (*M.N.*, vol. lx. p. 468). A closer and more satisfactory agreement could scarcely be hoped for.

R = 9^h 50^m $0^s{\cdot}72$ (207 rotations). This was a small and very brilliant and very definite spot, resembling a satellite when just entering on or leaving the planet's disc. It is shown in a diagram by L. dated 1880 July 9 (*Publications of the Potsdam Observatory*, No. 9, fig. 17), indenting the S. equatorial belt, and with a long, bright streak following it. On August 31 the preceding end of an elongated white spot was observed by W. to be in mid-transit at 11^h 35^m. It has been assumed that the middle of spot III. transited $2\frac{1}{2}^m$ later, or at 11^h $37^m{\cdot}5$. It seems not impossible that what was really observed was not the preceding *edge* of the oval spot, but the brighter spot at the preceding end of the oval spot, that is, spot III. itself. The longitude 19°·9 corresponding to this assumption would be in better agreement with the other observations. The question of the possible identity of this spot with either II. or IV. is considered under the latter spot.

White Spot IV.

Date. 1880.	G.M.T. of Transit. h m	Long.	Obs.	Date. 1880.	G.M.T. of Transit. h m	Long.	Obs.
Oct. 16	[9 14±]	[29°·1±]	W.	Nov. 11	4 45·2	26°·8	S.
,,	9 16·5	30·6	S.	12	10 19·7	29·3	S.
18	[10 25·2±]	[29·6±]	W.	15	7 3·7	24·1	S.
,,	10 27·4	31·0	S.	17	8 14	24·9	D.
,,	10 30	32·6	D.	,,	8 15	25·5	C.
19	6 1·9	27·7	S.	,,	8 15	25·5	W.
20	11 38	31·2	D.	,,	8 15·5	25·8	De.
21	7 11·7	27·3	S.	,,	8 17·1	26·8	S.
23	8 17·9	24·8	S.	19	9 20	22·1	C.
28	6 21·2	26·3	S.	,,	9 23	23·9	D.
,,	16 9·9	25·5	H.	,,	9 26·5	25·8	De.
29	11 53	27·2	W.	,,	9 28	27·0	M.
30	7 25	22·3	C.	20	5 0	21·9	C.
,,	7 28	24·1	D.	,,	5 2	23·1	D.
,,	7 29·7	25·1	S.	21	10 35	24·7	M.
Nov. 2	14 11·1	25·4	H.	,,	10 41	28·4	D.
,,	14 15	28·0	D.	22	6 15	30·6	M.
3	9 47	23·0	D.	,,	6 18	32·4	C.
4	5 25	21·6	D.	,,	16 5·2	24·5	B.
,,	5 28·1	23·5	S.	26	8 40	26·6	M.
6	6 37·1	22·6	S.	,,	8 43	28·5	G.
8	7 48	22·8	D.	,,	8 44	29·1	D.
,,	7 48·5	23·2	S.	27	4 19	25·8	D.
,,	17 44·2	26·4	H.	,,	4 21	27·1	W.
9	3 30·0	23·9	S.	28	9 49	25·6	M.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Nov. 28	9 51	26·8	D.	Dec. 24	5 36·5	28·7	M.
29	5 27	24·1	D.	25	11 17	34·7	W.
,,	5 27	24·1	M.	28	7 59	28·8	D.
Dec. 2	12 11·9	26·3	H.	29	13 28·2	27·9	B.
,,	12 14·2	27·7	B.	30	9 11	29·4	M.
3	7 54	27·4	D.	,,	9 15	31·8	D.
6	14 40·2±	30·2±	B.	31	4 47	27·6	M.
,,	14 42·0	32·6	H.	,,	14 41·5	29·3	H.
7	10 15·5	27·2	T.	1881.			
9	11 32·8	31·6	H.	Jan. 6	8 16·6	24·2	L.
10	7 14	31·5	D.	,,	8 26·5	30·2	M.
12	8 23	30·4	M.	,,	8 27	30·5	D.
,,	8 27	32·8	D.	7	4 5	29·0	D.
14	9 29·5	27·4	T.	,,	4 6	29·6	M.
15	5 16	31·3	D.	,,	13 49·7	25·7	B.
21	8 48	30·6	M.	13	7 37	27 8	D.
,,	8 49	31·2	D.	15	8 53	30·7	W.
22	4 26·5	29·4	L.	16	4 28	27·4	D.
23	10 4	33·6	D.	30	12 52·1	30·1	H.

Notes.—Oct. 16.—The following end of a long oval spot was observed to transit at $9^h\ 55^m$ (W.), and the time of the brighter spot at the preceding end has been inferred from a sketch. Oct. 18.—The middle of the above-mentioned long oval spot transited at $10^h\ 45^m$ (W.). The time of the white spot at the preceding end has been estimated from a sketch.

R = $9^h\ 50^m\ 7^s{\cdot}27$ (259 rotations). Kortazzi derived a rotation per. of $9^h\ 50^m\ 7^s$ from his observations made between 1880 November 15 and 1881 January 9;* and Hough one of $9^h\ 50^m\ 9^s{\cdot}8$ from his observations between 1880 October 26 and 1881 January 30. The spot was an exceedingly brilliant and definite† one, and was well observed, there being as many as 87 observations available. The motion of the spot, as is usually the case with the equatorial spots, was not quite uniform, though more so than is generally the case.

The question whether this spot is identical with spot III. or not is an important one. Marth left the question open. Hough considered them to be different, whilst Lohse thought there was only one spot, as to a certain extent he was justified in doing, because he had no observations between July 16 and December 22.

* The individual observations are not given in the only paper by Kortazzi known to the writer, so that they have not been included in the present list.

† But on the *f.* or *nf.* side it was not so well defined, the brighter material of the spot merging imperceptibly into the long oval spot following.

Kortazzi, too, considered that there was probably only a single spot. The nature of the problem is clearly shown by the diagram (p. 152). It is unfortunate that there should be no observations available in the first half of October, but, in the absence of any connecting link at this time, it seems impossible to treat them otherwise than as two separate spots. Mr. Denning, to whom the question was referred, is of the same opinion. He writes that "I should say that the two spots are different. The change of longitude, amounting to nearly 20°, seems too great between October 1 and 15 to be admitted in one identical object, though we know there are some such displacements. In any case, my opinion would be that the spots are different, and that the chief white spot [IV.] broke out into undue prominence at the middle of October 1880." It should be mentioned that the observations of W. show that there was much dark material in about long. 10° to 20° on October 16 and 18, so that it does not seem safe to identify spot II. with spot III., though they might quite possibly have been the same. Also Hough's observation of a white spot in long. 39°·3 on July 24 might be considered to be an early observation of spot IV., though here, again, it is impossible to be sure, owing to the long intervening gap. Spot IV. has been figured amongst others by Denning in *Monthly Notices*, vol. xli. p. 42; and by Lohse in *Publications of the Potsdam Astrophysical Observatory*, No. 9, taf. 36, fig. 10. Spot III. has also been figured by the last-mentioned observer in fig. 17, p. 11, of the work already cited. The two spots were very similar as regards their appearance.

White Spot V.

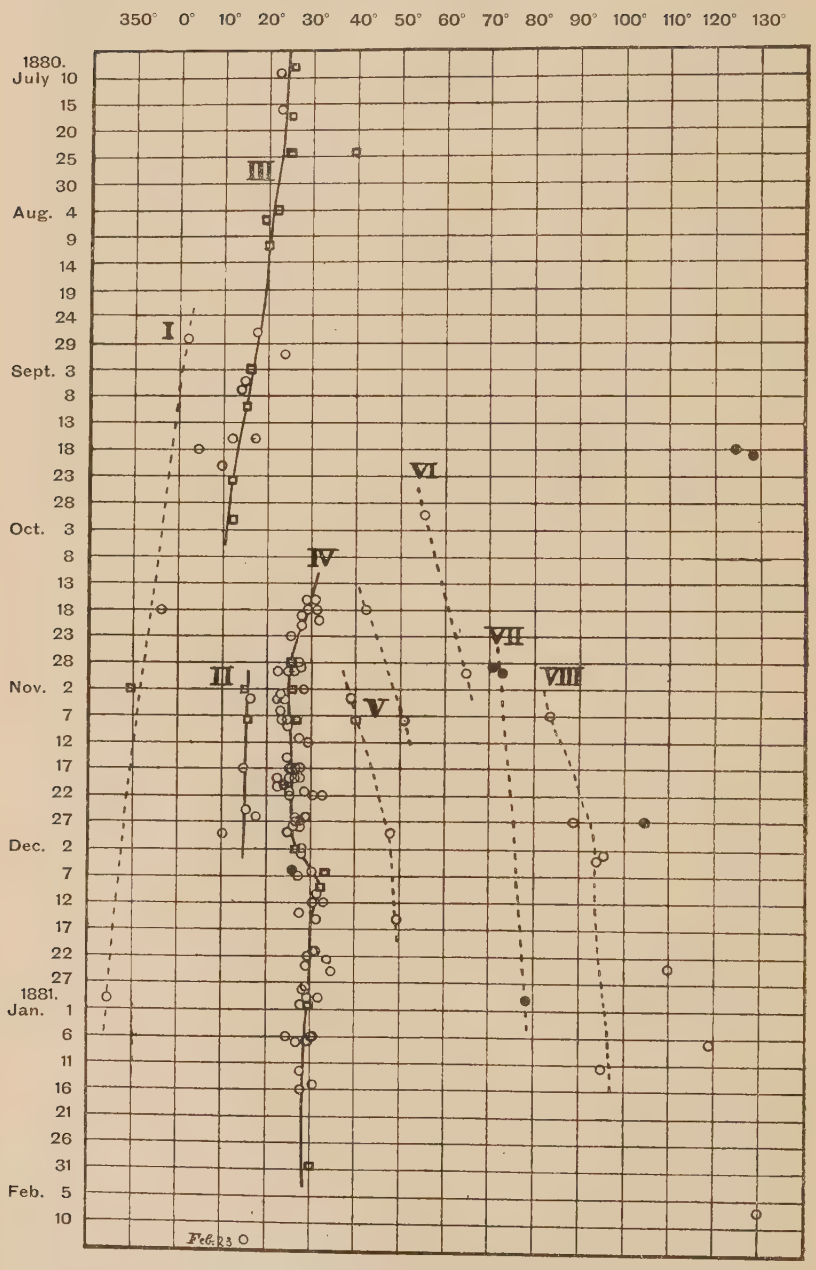
Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Oct. 18	[10 45]	[41·8]	W.	Nov. 8	[8 34·5]	[51·1]	W.
Nov. 4	5 53	38·7	D.	29	6 5	47·3	D.
8	18 6·8	40·2	H.	Dec. 15	5 46	49·6	D.

R = 9^h 50^m $18^s·67$ (100 rotations). The two observations by W. evidently relate to a different reference point to that observed by the other observers.

White Spot VI.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m		
Sept. 30	10 25	54·9	W.	Oct. 30	8 34	64·3	W.

The spot was also seen and drawn by W. on October 29 and by B. on November 7, so that although only two transit observations are on record the identification would appear to be certain. R = 9^h 50^m $20^s·16$ (73 rotations).



EQUATORIAL CURRENT OF JUPITER IN 1880.—A. STANLEY WILLIAMS.

Dark Spot VII.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Oct. 29	13 4	70·5	W.	Dec. 30	10 32	78·7	W.
30	8 47	72·3	W.				

The spot was also seen and drawn by B. on November 7, and the transit of the next spot, VIII., was recorded by him on the same night. Nevertheless, owing to the large gap in the observations, there is some doubt as to the certainty of the identification. $R = 9^h\ 50^m\ 12^s{\cdot}17$ (151 rotations).

White Spot VIII.

Date.	G.M.T. of Transit.	Long.	Obs	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Nov. 7	13 46·2	82·8	B.	Dec. 4	5 22·5	93·5	W.
27	6 3	89·2	W.	1881.			
Dec. 3	9 45	95·1	M.	Jan. 12	3 55	94·2	M.

$R = 9^h\ 50^m\ 14^s{\cdot}43$ (160 rotations). This was a very long spot, some 35° long, so that the observations are not very satisfactory, nor does the identification seem beyond doubt.

White Spot IX.

Date.	G.M.T. of Transit.	Long.	Obs.	Date.	G.M.T. of Transit.	Long.	Obs.
1880.	h m	°		1880.	h m	°	
Nov. 4	10 28	206·4	D.	Dec. 15	10 55	238·0	M.
8	13 3	214·9	D.	21	4 38	238·1	M.
12	5 43	220·6	D.	1881.			
23	7 19	221·9	D.	Jan. 3	7 40	247·0	W.
Dec. 14	15 7·2	233·5	B.	6	4 40	252·1	D.
15	10 58	239·9	D.	8	5 46	248·9	D.

Assuming that all the above observations really relate to the same feature, the rotation period would be about $9^h\ 50^m\ 32^s$ (158 rotations), which is considerably longer than that given by the other spots, and corresponds, indeed, with that indicated by the observations of late years. But this assumption is so precarious that there can be no hesitation in rejecting the result altogether, for the observations are certainly not all comparable. Thus, the last two relate to a break in the S. equatorial belt, whilst others relate to a white spot without any particular description, and the first one by M. to a "long white patch." Some of the unused observations in January 1881 are also suggestive of an alternative identification.

Unused Observations.

Date.	G.M.T. of Transit.	Long.	Obs.	Description.
1880.	h m	°		
July 24	19 19·7	39·3	H.	White spot.
Sept. 18	15 7	123·8	W.	Dark spot on S.E.B.
19	10 55	128·6	W.	,, ,,
Nov. 17	12 26 ±	153·7±	W.	,, ,,
20	8 30·7	150·4	N.	Large oval white spot.
,,	8 55	165·2	W.	Whitish spot.
27	6 27	103·8	W.	Dark spot on S.E.B.
Dec. 6	4 42	25·4	M.	,, ,,
10	11 42	163·4	W.	Whitish spot.
24	7 50±	110·1±	M.	Large white patch.
1881.				
Jan. 3	6 25	201·3	W.	Long, yellowish-white, ill-def. spot.
,,	7 21	235·4	W.	White spot, the *p.* and fainter of two.
7	6 33	119·3	W.	Moderately large bright spot.
12	6 29·5	188·4	W.	Small white spot.
,,	7 16·5	217·0	W.	Rather long white spot.
31	6 33	317·2	W.	Nearly round white spot.
Feb. 7	10 31·5	130·1	T.	Round bright spot.

The observations of the first eight spots are shown in diagram form on page 152 in order to indicate clearly at a glance both the certainty or otherwise of the identifications, and the degree of accordance of the individual observations. In this diagram the micrometer measures of the late Professor Hough have been shown by squares, in order to distinguish them from the other observations, which are indicated by circles. This distinction has been made in order to facilitate comparison between the micrometer observations and the eye-estimated observations. It will be seen that there is a most satisfactory agreement between the results obtained by the two methods. Whenever the eye-estimates show a departure from a straight line, there also the micrometer observations indicate a similar divergence. Indeed the micrometer observations cannot be said to show any superiority in point of accuracy over the eye-estimates. Spots III. and IV. are here particularly referred to, as the observations of other spots are too few in number to enable any conclusion to be drawn. If micrometer observations by several different observers had been available, it is certain that the measures of irregular markings, such as those on Jupiter, would have shown differences in the individual results of the various observers similar in character to those present in the eye-estimated observations, and probably quite as large.

As is not unusually the case, several observers mis-identified the spots observed by them, and derived anomalous values of

the rotation period. As an extreme example may be instanced Trouvelot, who concluded that the undermentioned observations all related to the same spot, and derived a rotation period of $9^h\ 51^m\ 8^s{\cdot}30$.

1880	Aug. 28	Long. =	1°·5
	Nov. 25	,, =	14·8
	Dec. 7	,, =	27·2
	14	,, =	27·4
1881	Feb. 7	,, =	130·1

The diagram shows clearly the impossibility of this identification, and that the five observations actually refer to four different spots.

Summary.

Spot.	R.	Wt.	Obs.	Rot.
	h m s			
I.	9 50 1·60	1	4	302
II.	6·73	1	8	65
III.	0·72	3	20	207
IV.	7·27	3	87	259
V.	18·67	1	6	100
VI.	20·16	½	2	73
VII.	12·17	1	3	151
VIII.	14·43	1	5	160
IX.	[32?]	...	11	158

The weights in the third column of the above table have been arbitrarily assigned, and, adopting them, the weighted mean rotation period of the equatorial current in the opposition of 1880–1 is

$9^h\ 50^m\ 7^s{\cdot}62$ (8 spots).

The weak point in this determination lies in the fact that the spots were all crowded together in one quadrant, instead of being distributed all round the planet. Also that in almost any system of weighting admissible, the two principal spots III. and IV. must have a large preponderating influence. Nevertheless, having regard to what we know relative to the motion of spot IV., and other equatorial spots in the succeeding two or three years, it seems safe to conclude that the rotation period of the current as a whole cannot have differed much from $9^h\ 50^m\ 7^s$. This is nearly 20^s shorter than the average value for the interval 1888 to 1908, equivalent to a difference in the velocity of the current of about 15 miles (24 km.) per hour. This appears to be a sufficiently remarkable fact to deserve attentive consideration, and to justify the present investigation, the conclusions derived from which may be summarised as under.

Conclusions.

(1) The rotation period of the Equatorial Current of Jupiter during the opposition of 1880–1 was $9^h\ 50^m\ 7^s{\cdot}62$, from 8 spots. This is nearly 20^s shorter than the value during the twenty years 1888–1908, and is equivalent to a real difference in velocity of about 15 miles (24 km.) per hour.

(2) The chief spots III. and IV. were really two distinct spots.

(3) That longitudes derived from eye-estimated transits are for all practical purposes equal in point of accuracy to those resulting from direct measures with a micrometer.

20 *Hove Park Villas, Hove:*
1910 *September* 29.

A preliminary paper on some quantitative Applications of Radiation Pressure to Cosmic Problems. By A. F. and F. A. Lindemann.

[Received 1910, June 4.]

According to the electro-magnetic theory of light, two black spheres of radius R_1, R_2, and absolute temperaratures T_1, T_2 at a distance ρ from one another repel each other with a force

$$\frac{\pi a}{4} \cdot \frac{R_1^2 R_2^2}{\rho^2} (T_1^4 + T_2^4)$$

where $\pi = 3{\cdot}141592\ \ldots$ and a is the constant of the Stefan-Boltzmann law of radiation

$$7{\cdot}061.10^{-15}\ \frac{erg}{cm^3\ degree^4}.$$

Here, as everywhere else in this paper unless specially stated, everything is measured in C.G.S. units. If the two bodies are not black, but have the albedoes A_1 and A_2, the force tending to drive them apart is

$$\frac{\pi a}{4} \cdot \frac{R_1^2 R_2^2}{\rho^2} \{(T_1^4 + T_2^4)(1 - A_1 A_2) + (T_1^4 - T_2^4)(A_2 - A_1)\}.$$

If the two bodies are simply particles in space without intrinsic heat of their own, the pressure they exert upon each other is

$$\frac{\pi R_1^2 R_2^2}{4\rho^2} \frac{E}{C} (2 + A_1 + A_2).$$

where E is the radiant energy which falls upon 1 cm^2 of each body in one second, and C is the velocity of light *in vacuo*. If only the energy falling upon the bodies from the Sun is taken into account, and the radiation from the planets be neglected,

$$E = \frac{aC}{4} \cdot \frac{R_0^2 T_0^4}{r^2},$$

where R_0 is the radius, T_0 the temperature, and r the distance from the Sun.

Taking R_0 as $6{\cdot}95 \times 10^{10}$cm, $T_0 = 6{,}000°$ abs. and $A_1 = A_2$,

$$p = \frac{a\pi}{8} \cdot \frac{R_0^2 R_1^2 R_2^2 T_0^4}{\rho^2 r^2} = 5{\cdot}523.10^{21} \frac{R_1^2 R_2^2}{\rho^2 r^2} .$$

It is assumed in this formula that the bodies radiate the heat they receive in all directions equally. If the densities of these two spheres are σ_1 and σ_2 the gravitational attractions will be

$$g = \frac{16\pi^2}{9} \cdot \frac{R_1^3 R_2^3 \sigma_1 \sigma_2}{\rho^2} \cdot 6{\cdot}685.10^{-7} .$$

Therefore $p = g$,—*i.e.* the spheres neither attract nor repel each other when

$$\frac{R_1 R_2 \sigma_1 \sigma_2}{1 + \frac{A_1 + A_2}{2}} = 1{\cdot}48.10^{-28} .$$

If A_1 is assumed equal to A_2, equal to 0·3, approximately the Moon's albedo, and if r is measured in units of the Earth's distance from the Sun instead of cm.'s,

$$R_1 R_2 \sigma_1 \sigma_2 r^2 = 85{\cdot}5.$$

For instance, if $\sigma_1 = \sigma_2 = 3$ and $r = 1$,

$$R_1 R_2 = 9{\cdot}5,$$

i.e. two stones of albedo 0·3 and of densities 3 will neither attract nor repel each other when they are at the same distance from the Sun as the Earth if the product of their radii is 9·5 cm.^{2}, or supposing them to be of equal size, if their diameters are 6·16 cm. If they are smaller they will repel each other, if larger they will attract each other.

This formula also shows that within the planetary system, that is, within 30 times the distance of the Earth from the Sun, only particles the product of whose density and radius in cm.'s is greater than 0·3 can attract one another. This does not apply, of course, to dust so fine that the diameter of the particles is very much smaller than the wave-length of the radiation pressing them apart.

If the Sun is considered a black body whose temperature is T and whose radius is R, it will exert the repulsive force

$$\frac{\pi a}{4} \cdot \frac{R_1^2 R_2^2}{r^2} \cdot T^4 \left(1 + A_2\right)\left(1 + \frac{R_1^2}{4r^2}\right)$$

upon a body at a distance r, whose radius is R_2 and whose albedo is A_2.

This will just balance the attraction when

$$R_2 \sigma_2 = 0{\cdot}831 \cdot 10^{-4}\left(1 + \frac{R_1^2}{4r^2}\right)$$

if the albedo of the particle is assumed to be 0·3 and its density σ_2. As the repelling force diminishes when R_2 is too small, *i.e.* when R_2 is smaller than $0{\cdot}25.10^{-4}$, σ_2 is probably smaller than 3. It is obvious that the second expression in the brackets only becomes of importance when r and R_1, *i.e.* when the distance of the particle from the Sun's centre and the Sun's radius are of the same order of magnitude. It is interesting, however, as it shows that a body can be held in equilibrium at a certain distance from the Sun without necessarily revolving round it. By means of this equation the minimum mass of the material composing the zodiacal light may be determined. Applying these formulæ to the theory that comets consist of meteoric particles, most of the well-known phenomena can be quantitatively treated. When far away from the Sun the temperature of the particles is low,

$$\frac{1{\cdot}118.10^9}{\sqrt{r}},$$

according to the above assumption that the Sun's temperature is 6000°; if, as is probably more correct, it is taken as 5880°, the temperature is

$$\frac{1{\cdot}096.10^9}{\sqrt{r}}.$$

The gravitational attraction therefore outweighs the radiation pressure when r is large and the particles are drawn together. As they approach the Sun the radiation pressure rises and they are driven apart. Any cosmic dust which they may have collected or formed by colliding with each other during their passage through space is driven off principally towards the Sun. As a general rule it cannot leave the comet in the opposite direction, as the sides of the particles in the shade are, firstly, colder, and therefore press less than the side which is exposed to the Sun; and secondly, there is no direct or reflected sunlight pressing the dust in this direction, but, on the contrary, a good deal of radiation pressing it in the opposite direction. The radiation pressure pressing one of the particles furthest from the nucleus towards the Sun is

$$\frac{K_1 F}{\rho^2 r^2} = \frac{K_2}{r^2},$$

where F is the surface of all particles in the head, ρ the distance of the particle in question from the centre of the nucleus, ρ is therefore the distance at which the pressure from the Sun just balances the pressure from the comet; K_1 and K_2 are constants.

It follows that $\rho^2 = K_3 F$, *i.e.* the brightness of the head is proportional to the square of the distance to which it projects towards the Sun from the centre of the nucleus. Possibly a careful consideration of the constant K_3 may enable one to determine the albedo as the pressure from the Sun is proportional to $1 + A$, whereas the pressure from the comet is proportional to $A(1 + A)$. This is due to

the fact that the particles in question are probably so small that the heat absorbed by the other particles, and then re-emitted with a very much greater average wave-length, do not cause any appreciable radiation pressure upon them. A similar consideration leads to the possibility of determining the albedo from the mutual pressure of the particles in the tail. This may be fairly accurately determined from the angle at which the particles move apart. The tail is of course conposed on this theory of small particles of dust, not as in Bredichin's theory of gases. It is difficult to see how radiation pressure should impart such large velocities to gases, besides which the tail could hardly keep such a well-defined shape at such high temperatures as one has to deal with. It is obvious that particles of various densities will form tails at angles to one another, as the masses of two particles will be in inverse ratio to the squares of the densities if they are of such a size that the same radiation pressure acts on both of them. If the repulsion from the comet is greater than that from the Sun, a tail pointing towards the Sun may be formed, though this can seldom be very long. The presence of cyanogen and of carbohydrates is accounted for by the possibility of these substances, which must exist chiefly in the solid state, outside the orbit of Jupiter, being collected on the surface of the particles composing the comet, like any other dust, and evaporating as they approach the Sun. Whether these gases are caused to fluoresce by the intense ultra-violet radiation falling upon them, or whether their spectrum is due to electric discharges occasioned by the slight friction of the dust against the larger particles, is an open question.

If the head of the comet does not rotate upon an axis of its own, it is possible to determine the average size of the particles.

Presuming the comet to be stable, *i.e.* not to change very much in the course of an orbit, the path that the average particle covers towards the centre of gravity of the comet in consequence of the gravitational attraction during the colder parts of its orbit must be equal to the path covered by the particles in the opposite direction, in consequence of the radiation pressure.

If N is the number of particles per cm^3, which may be expressed as a function of the average radius R of a particle if the brightness is known, ρ their distance from the centre of gravity, and r their distance from the Sun, expressed as the function $f(t)$ of the time which has elapsed since their passage through the perihelion,

$$\frac{d^2\rho}{dt^2} = \rho N R \left\{ 1{\cdot}076.10^{-6}\, R^2\sigma - \frac{1\ 736.10^{22}}{\sigma f(t)^2} \right\}.$$

If this equation is solved, and a time t determined so that the integral from o to t is equal to the integral from t to t_1, where t_1 is the period of half a revolution round the Sun, $R\sigma$ may be determined. For Halley's comet a rough approximation gives

$$R\sigma = 1{\cdot}3.$$

This result is of particular interest, as the mass may be calculated

from it, always assuming the head not to rotate. When Halley's comet was discovered in 1909 it was reported to have been at a distance of 10^9 kilometres, and to have been of magnitude 16·5. Assuming an albedo of 0·3 for the particles, and that the comet shines only with reflected sunlight, this leads to a surface of

$$1{\cdot}17.10^{14} cm^2.$$

The mass $m = \frac{4\pi}{3} R^3 \sigma N$, where N is the total number of particles and

$$N = \frac{1{\cdot}17.10^{14}}{\pi R^2}$$

or

$$m = \tfrac{4}{3} R\sigma.1{\cdot}17.10^{14}.$$

Introducing the above value of $R\sigma = 1{\cdot}3$,

$$m = 2{\cdot}03.10^{14} \text{ grammes},$$

or 203 million metric tons.

Naturally, if the head of the comet is in rotation, $R\sigma$, and consequently m, will be larger. If the diameter of the head is equal to that of the Moon, and the density of the particles equal to that of the Moon, *i.e.* 3·3, the brightness of a star passing behind the head will be weakened by about $\frac{1}{2000}$ of a magnitude, and the particles will be on an average 90 m. apart. The mass of the comet can on hardly any supposition be larger than about 40 billion tons, which would be the mass if the comet consisted of one piece of material of density 3·3. This is, of course, quite impossible, as it would preclude the possibility of seeing stars through the head of the comet at all. The mass of the tail can be approximately determined in a similar manner if its brightness is known. For instance, if it were found that when the tail's mean distance from the Earth was 40 million kilometres, and from the Sun 150 million kilometres, it reflected altogether as much light as is received from a first magnitude star, its surface would be $6{\cdot}1.10^{14}$ assuming an albedo of 0·3, and its mass

$$m = \tfrac{4}{3} R\sigma.6{\cdot}1.10^{14} = 6{\cdot}6.10^{10} \text{ grammes},$$

or about 66,000 metric tons.

As no measurements were available, these purely imaginary examples have been carried out to give some idea of the order of magnitude of the probable mass; but of course it will be easy to put in the correct figures so soon as they have been determined by careful observation.*

Assuming the average velocity of the particles in the tail to be 50 kilometres away from the Sun, and its length under the above conditions to be 80 million kilometres, the comet's mass would

* Since writing the above, exact data have been obtained, which we are, however, unable to utilise, as it is not desirable to make any alteration in the paper as sent in on June the 4th.

diminish by about 41 kilograms per second, or about 3500 tons a day about the time of the perihelion passage; in other words, as long as the tail continued as bright as assumed above. Possibly this diminution of the mass, which is probably greatly exaggerated in the above example, might produce a slight deviation from the true elliptical orbit. The measurements show, as was to be expected, that the effective surface grows as the distance from the Sun grows smaller, *i.e.* as the dust is more rapidly thrown off the particles in consequence of their increased temperature. This also would produce a disturbance in the orbit. It appears that the surface on which the Sun's pressure acts increased in Halley's comet approximately in the ratio $\frac{1}{r^2}$. Assuming this to be the case, the force acting between the Sun and the comet is

$$\frac{m_1 m_2}{r^2} \cdot 6{\cdot}685.10^{-8} - \frac{1{\cdot}444.10^{22} f}{r^4}$$

$$= \frac{1{\cdot}27.10^{26} m_2}{r^2} - \frac{1{\cdot}444.10^{22} f}{r^4}.$$

if m_2 is the comet's mass, r its distance from the Sun, and f a constant. It is obvious that if this equation is correct the comet will not move round the Sun in an ellipse, but in an oval curve, with its point away from the Sun. A comet revolving round the Sun in this curve would pass through its perihelion slightly later than if its orbit were an ellipse. This appears to have been the case in 1910. A calculation shows that the radiation pressure on a comet of magnitude 1 at a distance of 120 million kilometres from the Sun and 40 million from the Earth, whose mass was 200 million tons, would be 0·064 per cent. of the gravitational attraction.

If the surface $F = f(r)$ is determined exactly as a function of the distance from the Sun, the formula for this curve can be developed, and the mass calculated from the observed orbit.

For this purpose the brightness of the comet's head over as long a stretch of its orbit as possible should be measured, and an empirical formula for the surface developed.

A large number of other phenomena may be treated from this point of view, and it is hoped that they may form the subject of a future paper. It was considered, however, that it might be of interest to publish these results immediately, however incomplete, in view of the propinquity of Halley's comet, and the exceptional facilities offered thereby for testing them.*

* See footnote, previous page.

Sidmouth and Berlin:
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